RETAINING-WALLS
FOR EARTH.

INCLUDING

THE THEORY OF EARTH-PRESSURE AS DEVELOPED FROM THE ELLIPSE OF STRESS.

WITH

A SHORT TREATISE ON FOUNDATIONS, ILLUSTRATED WITH EXAMPLES FROM PRACTICE.

BY

MALVERD A. HOWE, C.E.,

Professor of Civil Engineering, Rose Polytechnic Institute;
Member American Society of Civil Engineers.

THIRD EDITION, REVISED AND ENLARGED.

FIRST THOUSAND.

NEW YORK:
JOHN WILEY & SONS.
LONDON: CHAPMAN & HALL, LIMITED.
1896.

ROBERT DRUMMOND, ELECTROTYPER AND PRINTER, NEW YORK.

CONTENTS.

THEORY OF EARTH-PRESSURE.

FORMULAS FOR EARTH-PRESSURE.

Recapitulation.

EXAMPLES.

FOUNDATIONS.

FOUNDATIONS UNDER WATER AND DEEP FOUNDATIONS.

TABLES.

PREFACE TO THE SECOND EDITION.

THE first edition of this work was based upon the theory advanced by Prof. Weyrauch in 1878, but owing to the length of the demonstrations used by him, it was thought advisable to present different and shorter demonstrations in this edition. To show that the new demonstrations give identical results with those obtained by Prof. Weyrauch, his demonstrations have been given in an appendix as they appeared in the first edition.

The new demonstrations are based upon the theory first advanced by Prof. Rankine in 1858. Those readers who are familiar with Rankine's Ellipse of Stress can omit pages 1 to 9, inclusive, in following the demonstrations.

An attempt has been made to present the theory in a shape easily followed by those who have only a knowledge of algebra, geometry, and trigonometry; whenever calculus has been resorted to, the work has been simplified as much as possible. For convenience in practice, the formulas have been arranged in a condensed shape in Part I, and are followed by numerous examples illustrating their application.

The values of various coefficients have been computed and tabulated and will be found to very materially decrease the labor of substitution in the formulas.

It is hoped that the introduction of a brief treatment of the supporting power of earth in the case of foundations, as well as the formula for determining the breadth of the base of a retaining-wall, will prove acceptable.

For valuable help in the verification of proofs of formulas, and the critical reading of the whole text, I acknowledge the kind assistance of Prof. Thos. Gray.

M. A. H.

PREFACE TO THE THIRD EDITION.

In this edition a large number of examples illustrating several profiles of retaining-walls and types of foundations selected from existing structures have been included. The Appendix of the second edition has been replaced by a treatise on Foundations sufficiently short and, the author believes, sufficiently complete for the use of technical schools, if judiciously supplemented by lectures or references to descriptions of existing structures.

M. A. H.

TERRE HAUTE, IND., NOV. 1896.

NOMENCLATURE.

ϕ = the angle of repose, or the maximum angle which any force acting upon any plane within the mass of earth can make with the normal to the plane.

ϵ = the angle made by the surface of the earth with the horizontal; ϵ is *positive* when measured *above* and *negative* when measured *below* the horizontal.

α = the angle which the back of the wall makes with the vertical passing through the heel of the wall; α is *positive* when measured on the *left* and *negative* when measured on the *right* of the vertical.

δ = the angle which the direction of the resultant earth-pressure makes with the horizontal.

ϕ' = the angle of friction between the wall and its foundation.

ϕ'' = the angle of friction between the back of the wall and the earth.

H = the vertical height of the wall in feet.

h = the depth of earth in feet which is equivalent to a given load placed upon the surface of the earth.

B' = the width in feet of the top of the wall.

B = the width in feet of the base of the wall.

Q = the distance in feet from the toe of the wall to the point where R cuts the base.

P = the resultant earth-pressure in pounds against a vertical wall.

E = the resultant earth-pressure in pounds against any wall.

R = the resultant pressure in pounds on the base of the wall.

G = the total weight in pounds of material in the wall.

γ = the weight in pounds of a cubic foot of earth.

W = the weight in pounds of a cubic foot of wall.

p = the intensity of the pressure in pounds on the base of the wall at the toe.

p' = the intensity of the pressure in pounds on the base of the wall at the heel.

p_0 = the average intensity of the pressure in pounds on the base of the wall.

$x = H \tan \alpha$.

x'' and x' = depth of the base of the foundation below the earth surface.

B'' = breadth of the base of the foundation.

o = the offset of a foundation course.

G' = the total weight of the material above the base of the foundation.

THEORY OF EARTH-PRESSURE.

Preliminary Principles.—Before demonstrating the general formula for the thrust of earth against a wall, it will be necessary to establish the relations between the stresses in an unconfined and homogeneous granular mass.

* In Fig. 1 let ABC be any small prism within a granular mass which is in equilibrium un er the action of the three stresses P, Q, and R, having the intensities p, q, and r respectively.

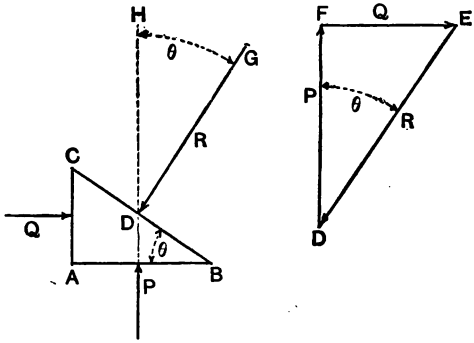


FIG. 1.

* In all the demonstrations which follow, the dimension perpendicular to the page will be considered as unity.

Let θ represent the angle of inclination of the plane CB with AB, and the angle at A be a right angle.

The planes AB and AC are called planes of principal stress, and P and Q are called principal stresses.

CASE I. *If the principal stresses are of the same kind and their intensities the same, then will the resultant stress on any third plane be normal to that plane and its intensity be equal to that of either principal stress.*

In Fig. 1, for convenience, let $AB = 1$, then $AC = \tan\theta$, and $CB = \frac{1}{\cos\theta}$. Hence

$$P = p,\ \ Q = q\tan\theta = p\tan\theta, \text{ since } p = q, \text{ and } R = \frac{r}{\cos\theta}.$$

Since P, Q, and R are in equilibrium, they will form a closed triangle, as shown on the right in Fig. 1. Hence

$$R^2 = P^2 + Q^2,$$

or

$$\frac{r^2}{\cos^2\theta} = p^2 + p^2\tan^2\theta = p^2(1 + \tan^2\theta);$$

$$\therefore\ r = p = q.$$

Also,

$$R\cos FDE = P,$$

or

$$\frac{r}{\cos\theta}\cos FDE = p; \quad \text{but } r = p.$$

Hence

$$\cos\theta = \cos FDE = \cos HDG;$$

$$\therefore\ HDG = \theta \quad \text{and} \quad R \text{ is normal to } CB.$$

CASE II. *If the principal stresses are not of the same kind but their intensities the same, then will the resultant make the angle θ with the direction of the principal stress, but on the opposite side from that on which the resultant in Case* I *lies, and its intensity be equal to that of either principal stress.*

The demonstration of Case I proves this principle if Fig. 1 is replaced by Fig. 2.

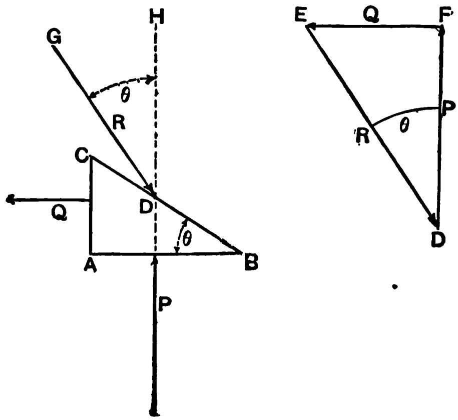


FIG. 2.

CASE III. *Given the principal stresses of the same kind but having unequal intensities, to determine the intensity and direction of the resultant stress on any third plane.*

Let P and Q be compressive and the intensity $p >$ the intensity q.

The following identities can be written:

$$p = \tfrac{1}{2}(p + q) + \tfrac{1}{2}(p - q),$$

and

$$q = \tfrac{1}{2}(p + q) - \tfrac{1}{2}(p - q);$$

or the resultant intensity on the plane CB may be considered as being the resultant of two intensities, one being the intensity of the resultant stress caused by two like principal stresses having the same intensity $\frac{1}{2}(p+q)$, and the other the intensity of the resultant stress caused by two unlike principal stresses having the same intensity $\frac{1}{2}(p-q)$.

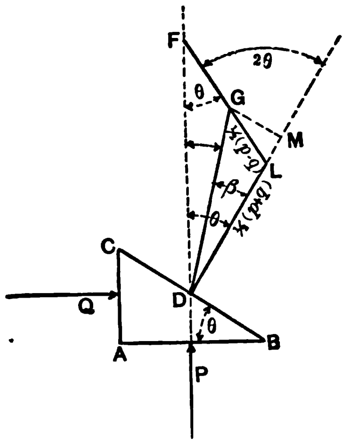


FIG. 3.

The intensity of the resultant stress caused by the first two principal stresses will be, by Case I, $\frac{1}{2}(p+q)$, and the direction of the resultant will be normal to the plane CB. By Case II the resultant of the second pair of principal stresses will make the angle θ with the direction of P, and its intensity will be $\frac{1}{2}(p-q)$; then the resultant intensity can be found as follows:

In Fig. 3 draw MD normal to BC, and make $LD = \frac{1}{2}(p+q)$; with L as a centre and LD as radius, describe an arc cutting FD at F. Then the angle $LFD = LDF = \theta$. Lay off $LG = \frac{1}{2}(p-q)$, and draw GD, which is the result-

ant intensity, and the intensity of the resultant stress on CD caused by the two principal stresses P and Q. GD also represents the direction of the resultant stress R.

Since the intensities of the principal stresses remain constant, $\frac{1}{2}(p+q)$ and $\frac{1}{2}(p-q)$ will remain the same for any inclination of the plane CB; hence the intensity r of the resultant depends upon the angle θ when p and q are given.

From Fig. 3,

$$GL \cos 2\theta = LM \quad \text{and} \quad GL \sin 2\theta = GM,$$

$$DM = DL + LM = \tfrac{1}{2}(p+q) + \tfrac{1}{2}(p-q)\cos 2\theta,$$

$$\overline{GD}^2 = r^2 = \overline{GM}^2 + \overline{DM}^2,$$

or

$$r = \sqrt{p^2 \cos^2 \theta + q^2 \sin^2 \theta}, \quad . \quad . \quad . \quad . \quad (a)$$

which is the general expression for the intensity of the resultant stress of a pair of principal stresses.

As the angle θ changes, the angle β will also change, and it will have its maximum value when the angle $LGD = 90°$. This is easily proven as follows:

With L as centre and GL as radius describe an arc; then β will have its maximum value when the line DG is tangent to the arc; but when DG is tangent to the arc the angle LGD is a right angle, since LG is the radius of the arc.

$$\sin \max \beta = \frac{p-q}{p+q}, \quad . \quad . \quad . \quad . \quad . \quad (b)$$

from which the following can be easily obtained:

$$\frac{p}{q} = \frac{1 + \sin \max \beta}{1 - \sin \max \beta}, \quad . \quad . \quad . \quad . \quad . \quad (c)$$

which expresses the limiting ratio of the intensities of the principal stresses consistent with equilibrium, p being greater than q.

CASE IV. *Given the intensity and direction of the resultant stress on any plane, and the value of max β, to determine the intensities and directions of the principal stresses.*

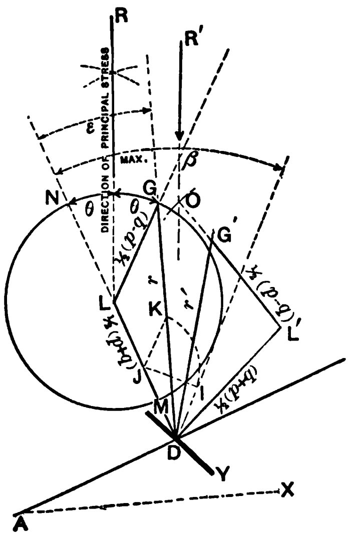


FIG. 4.

Let AD represent the given plane and GD the direction and intensity of the resultant stress at the point D.

Draw DL normal to AD, and draw DI, making the angle max β with LD. At any point J in DL describe an arc tangent to DI, cutting GD in K and draw GL parallel to KJ; with L as a centre and LG as radius describe

a circumference. This circumference will pass through G and be tangent to DI; hence $\frac{GL}{DL} = \sin \max \beta$.

Since $\sin \max \beta = \frac{p-q}{p+q}$, and GL and LD are components of r,

$$GL = \tfrac{1}{2}(p-q) \quad \text{and} \quad DL = \tfrac{1}{2}(p+q);$$

then $ND = NL + LD = \frac{1}{2}(p-q) + \frac{1}{2}(p+q) = p,$

and $MD = LD - LM = \frac{1}{2}(p+q) - \frac{1}{2}(p-q) = q,$

which completely determines the intensities of the principal stresses.

According to Case III, the direction of the greater principal stress bisects the angle between the prolongation of LM and the line GL; hence RL represents the direction of the greater principal stress, and that of the other is at right angles to RL.

The above intensities and directions being determined, the intensity of the resultant stress on any other plane passing through D is easily determined as follows:

Let DY represent any plane passing through D, draw DL' normal to DY and equal to $\frac{1}{2}(p+q)$. Draw $R'D$ parallel to RL, and with L' as a centre and $L'D$ as radius describe an arc cutting $R'D$ at O, and make $L'G' = \frac{1}{2}(p-q)$; then $G'D = r' =$ the intensity of the resultant stress on DY.

It is clear that if the value of max β can be obtained for a mass of earth that the construction of Fig. 3 can be employed in determining the intensity of the earth-pressure at any point in *any plane* within the mass.

It has been established by experiment that if a body be placed upon a plane, that (as the plane is made to incline to the horizontal) at some angle of inclination the body will commence to slide down the plane, and that this angle depends largely upon the *character* of the surfaces in contact.

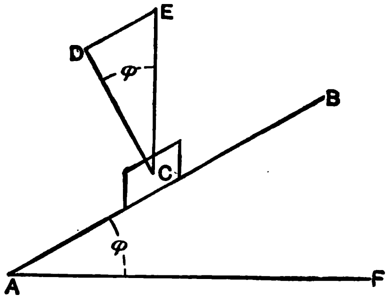


FIG. 5.

In Fig. 5 let AB represent a plane inclined at the angle ϕ with the horizontal, and C any mass just on the point of sliding down the plane. Let EC represent the weight of the mass C, and ED and DC the components respectively parallel and normal to the plane AB. Then DE is the force required to just keep the mass C from sliding down the plane, assuming the plane to be perfectly smooth, or if the plane is rough this force represents the effect of friction.

$$\frac{DE}{DC} = \tan\ \phi,$$

or when the mass C is about to slide, the resultant pressure EC on AB makes the angle ϕ with the normal to the

plane, the angle ϕ being the inclination of the plane AB, and is called the angle of friction.

In the case of earth, considered as a dry granular mass, the inclination of the steepest plane upon which earth will not slide is called the angle of repose, and the plane the surface of repose.

From the above, then, it follows that in a mass of earth the resultant pressure on any plane cannot make an angle with the normal to that plane which is greater than the angle of repose ϕ; therefore the construction of Case IV applies to earth when max β is replaced by ϕ. The values of ϕ for earth under various conditions are given in Table II.

The preceding principles will now be applied in determining the thrust of earth against a retaining-wall.

EARTH-PRESSURE.

In order that the formulas may not become too complex for practical use, it will be assumed that the earth is a homogeneous granular mass without cohesion. The surface of the earth will be considered to be a plane, and the length of the mass measured normally to the page as unity.

* *Given the intensity and direction of the resultant stress at any point in any plane parallel to the surface of the earth, the inclination of the surface of the earth with the horizontal, and the angle of repose, to determine the intensity and direction of the resultant stress on a vertical plane passing through the same point.*

---

* For comparison, see the "Technic," 1888; a construction by Prof. Greene.

The construction follows (see Fig. 4, above) directly from Rankine's Ellipse of Stress.

In Fig. 6 let BQ represent the surface of the earth, and D any point in the plane AD parallel to BQ; draw DQ normal to AD, and make the vertical GD equal to QD; then $GD \cdot \gamma$ is the intensity of the resultant pressure at D. Draw DM, making the angle ϕ with LD, and with L as centre describe an arc tangent to DM and passing through G; then by Case IV $LG \cdot \gamma = \frac{1}{2}(p - q)$, $LD \cdot \gamma = \frac{1}{2}(p + q)$,

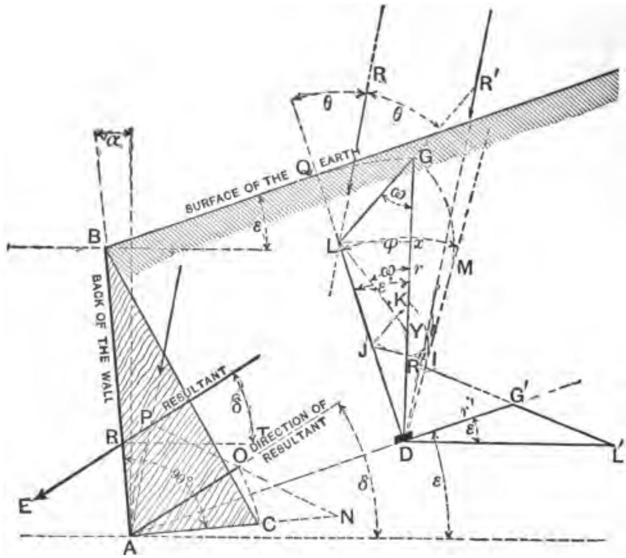


FIG. 6.

and RL bisecting the angle QLG is the direction of the greater principal stress. To determine the intensity and direction of the resultant stress at D on a vertical plane, proceed according to Case IV. Draw $R'D$ parallel to RL and $DL' = DL$ normal to DG. With L' as a centre and $L'D$ as radius describe an arc cutting $R'D$ at R'', and make

$L'G' = LG$; then DG' represents the direction of the resultant stress, and $DG' \cdot \gamma$ the intensity of the resultant.

In Fig. 6 the angle $R'DL' = DR''L' = 90° - \omega + \theta$. $\therefore\ G'L'D = 2\omega - 2\theta$. But $2\theta = \omega + \epsilon$; hence $G'L'D = \omega - \epsilon$.

Draw $LY = LG$; then the angle $DLY = \omega - \epsilon$. $\therefore$ Since $LD = DL'$ and $LY = LG = L'G'$, the triangle $G'L'D$ equals the triangle LYD and the angle $G'DL' = \epsilon$; or *the direction of the resultant earth-pressure against a vertical plane is parallel to the surface of the earth.*

From Fig. 6,

$$\tfrac{1}{2}(p - q) \cos \omega = GX \cdot \gamma,$$

$$\tfrac{1}{2}(p - q) \sin \omega = LX \cdot \gamma,$$

$$\tfrac{1}{2}(p + q) \cos \epsilon = DX \cdot \gamma.$$

Now $$DY = DG' = DG - 2GX,$$

or

$$DG' \cdot \gamma = DG \cdot \gamma - (p - q) \cos \omega$$

$$= \tfrac{1}{2}(p + q) \cos \epsilon - \tfrac{1}{2}(p - q) \cos \omega,$$

$$\tfrac{1}{2}(p + q) : \sin \omega \quad :: \quad \tfrac{1}{2}(p - q) : \sin \epsilon,$$

and

$$\sin \omega = \frac{p + q}{p - q} \sin \epsilon,$$

or

$$\cos \omega = \sqrt{1 - \left(\frac{p + q}{p - q}\right)^2 \sin^2 \epsilon} = \sqrt{\frac{(p-q)^2 - (p + q)^2 \sin^2 \epsilon}{(p - q)^2}},$$

and since $$\tfrac{1}{2}(p + q) \sin \phi = \tfrac{1}{2}(p - q),$$

$$\cos \omega = \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi}.$$

Substituting this value for $\cos \omega$ in the equation for $DG' \cdot \gamma$, it becomes

$$DG' \cdot \gamma = \tfrac{1}{2}(p + q) \cos \epsilon - \tfrac{1}{2}(p - q) \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi},$$

or since $$\frac{1}{\sin \phi} = \frac{p + q}{p - q},$$

$$DG' \cdot \gamma = \tfrac{1}{2}(p + q) \{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}\}.$$

In a similar manner,

$$DG \cdot \gamma = \tfrac{1}{2}(p + q) \{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}\},$$

and

$$\frac{DG'}{DG} = \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}};$$

hence

$$DG' = DG \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}.$$

Let $x =$ the *vertical* distance between the two planes BQ and AD, then

$$DG = DQ = x \cos \epsilon.$$

$$\therefore DG' \cdot \gamma = (x) \gamma \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}},$$

which is the expression for the intensity of the resultant earth-pressure on a vertical plane at any depth x below the surface.

Let

$$* \; A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}. \quad . \quad . \quad (d)$$

* See Rankine's Applied Mechanics; Alexander's Applied Mechanics; Theories of Winkler and Mohr.

The average intensity of the resultant earth-pressure on a vertical plane of the length x will be

$$\left(\frac{x}{2}\right)\gamma A,$$

and hence the total pressure will be

$$P = \frac{x^2}{2}\gamma A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

Since the intensities of the pressures are uniformly varying from the surface, and increasing as x increases, the application of the resultant thrust will be at a depth of $\frac{2}{3}x$ below the surface.

Considering the earth as an unconfined mass, the above formula is perfectly general and can be applied under all conditions, including the case when ϵ is negative.

The resultant stress on any plane as AB, Fig. 6, can be found by applying the principles of Case IV. Draw PA parallel to RL, make $AN = LD$ and $NO = LG$; then AO represents the direction of the resultant pressure on AB. Make $AC = AO$; then the area of the triangle ABC multiplied by γ is the total pressure on the plane AB, and this pressure is applied at $\frac{2}{3}AB$ below B.

In unconfined earth this construction is perfectly general and applies to *any plane.* It also applies equally well to curved profiles. An example illustrating the application of the method will be given in the *applications.* See pages 22 and 23.

The following graphical construction, Fig. 7, is more convenient than that of Fig. 6.

As before, let BE represent the surface of the earth, and

AD a plane parallel to the surface. At any point D in this plane, draw DE vertical and make $DF = DE$; draw FG horizontal and make the angle $HFD = \phi$.

With L as a centre, describe an arc passing through G and tangent to MF; then with L as a centre and LF as

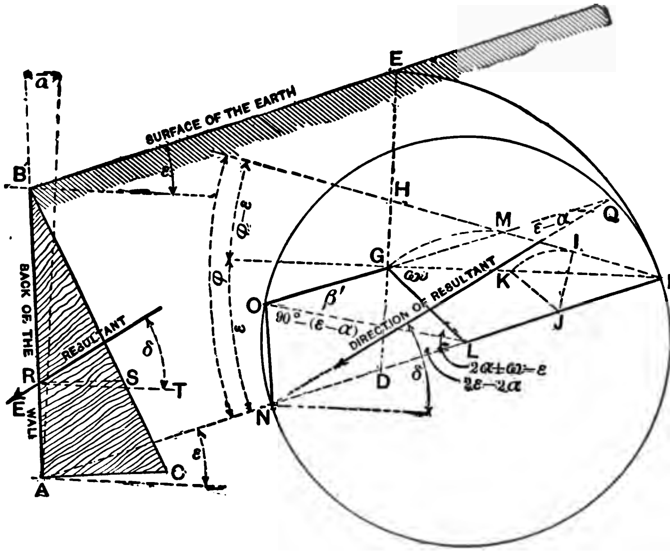


FIG. 7.

radius, describe the circumference FON, cutting AD at N; through N draw NO parallel to AB, then draw AC normal to AB and equal to OG. The area of the triangle ABC multiplied by γ will be the total earth-pressure on AB. To determine the direction of the thrust prolong OG to Q, then QN is the direction of the thrust.

That this construction is equivalent to that of Fig. 6 is

proved as follows. The triangle GLF of Fig. 7 equals the triangle GLD of Fig. 6.

$$\therefore\ GL \cdot \gamma = \tfrac{1}{2}(p-q) \quad \text{and} \quad LF \cdot \gamma = LO \cdot \gamma = \tfrac{1}{2}(p+q).$$

In Fig. 6, the angle $NAP = NPA = 90° - \tfrac{1}{2}(\omega - \epsilon) - \alpha.$

$$\therefore\ ONA = \omega - \epsilon + 2\alpha.$$

In Fig. 7, the angle $OLN = 2\epsilon - 2\alpha$. But $GLN = \omega + \epsilon$.

$$\therefore\ GLO = \omega - \epsilon + 2\alpha,$$

and GO of Fig. 7 equals AO of Fig. 6.

In Fig. 7, the angle $QNO = 90° - \beta'$.

In Fig. 6, the angle $OAB = 90° - \beta'$.

Therefore the direction of the thrust is the same in both constructions.

The two constructions given above are all that is required to determine the thrust of earth upon any plane within the mass of earth, as one can be used as a check upon the other; but as a formula is often very convenient, a general formula will now be deduced which will enable one to determine the values of E and δ for any plane within a mass of earth.

General Formula for the Thrust of Earth.

In Fig. 8, let BQ represent the surface of the earth and AB any plane upon which the earth-pressure is desired.

Draw AD parallel to BQ and let the vertical distance $QD = FA = x$,

From (*e*) the earth-pressure upon FA is parallel to the surface and equal to

$$P = \frac{x^2}{2}\gamma A.$$

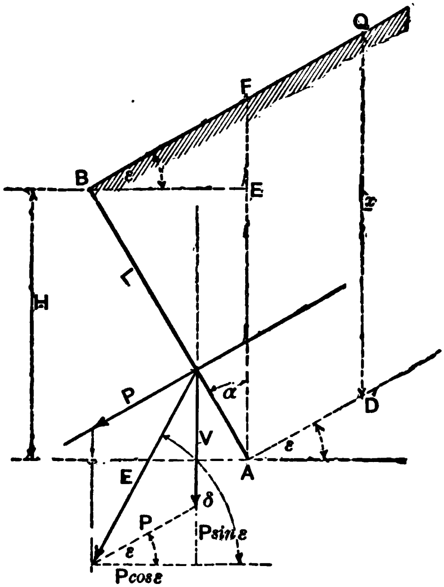


FIG. 8.

$$\text{But } AF = x = H(1 + \tan\alpha\tan\epsilon) = H\frac{\cos(\epsilon-\alpha)}{\cos\alpha\cos\epsilon};$$

$$\therefore\ P = \frac{H^2\gamma}{2}\frac{\cos^2(\epsilon-\alpha)}{\cos^2\alpha\cos^2\epsilon}A. \quad . \quad . \quad . \quad (f)$$

Now the thrust P combined with the weight of the prism ABF must produce the resultant pressure upon AB.

Then from Fig. 8,

$$V = \frac{H^2\gamma}{2} \tan \alpha \,(1 + \tan \alpha \tan \epsilon)$$

$$= \frac{H^2\gamma}{2} \frac{\sin \alpha \cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon}, \quad . \quad (g)$$

$$E = \sqrt{(V + P \sin \epsilon)^2 + (P \cos \epsilon)^2} = \sqrt{V^2 + P^2 + 2\,VP \sin \epsilon}.$$

Substituting (f) and (g) in this it becomes

$$E = \frac{H^2\gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times$$

$$\sqrt{\sin^2 \alpha + 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \frac{A}{\cos \epsilon} + \cos^2 (\epsilon - \alpha) \frac{A^2}{\cos^2 \epsilon}},$$

which becomes, by replacing A by its value from (d),

$$E = \frac{H^2\gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times$$

$$\sqrt{\begin{array}{l} + \sin^2 \alpha \\ + 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \dfrac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \\ + \cos^2 (\epsilon - \alpha) \left\{ \dfrac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}^2 \end{array}}, \quad . \quad (1)$$

which is the general equation for the thrust of earth upon *any plane* within the mass.

To determine the direction of the thrust of the earth, let δ be the angle which the direction of the thrust makes with the horizontal; then, from Fig. 8,

$$\tan \delta = \frac{V}{P \cos \epsilon} + \tan \epsilon,$$

Substituting the values of V and P given above, this becomes

$$\tan \delta = \frac{\sin \alpha \cos \epsilon + \sin \epsilon \cos (\epsilon - \alpha) A}{\cos \epsilon \cos (\epsilon - \alpha) A}, \quad . \quad (1a)$$

where

$$A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}. \quad . \quad . \quad (d)$$

Equations (1) and (1a) are readily reduced to more simple forms for special cases. These forms will be found in Part I.

The Plane of Rupture.—Although it is not necessary to know the position of the plane of rupture in order to determine the thrust of the earth, yet it may be of interest to know its position, which can be easily determined as follows:

The plane of rupture will be back of the wall and pass through the heel of the wall. The resultant earth-pressure will make the angle ϕ with the normal to this plane. Now the tangent of the angle which the direction of the resultant earth-pressure on any plane makes with the horizontal is determined from the formula

$$\tan \delta = \frac{\sin \alpha}{\cos (\epsilon - \alpha) A} + \tan \epsilon.$$

If ω represents the angle which the plane of rupture makes with the vertical passing through the heel of the wall, $\alpha = \omega$ and $\delta = \phi + \omega$.

$$\tan (\phi + \omega) = \frac{\sin \omega}{\cos (\epsilon - \omega) A} + \tan \epsilon,$$

from which the value of ω can be determined for any case.

For the case where $\epsilon = \phi$, ϵ being positive with respect to the wall and *negative with respect to the plane of rupture*, the above equation becomes

$$\tan(\phi + \omega) = \frac{\sin \omega}{\cos(\phi + \omega)\cos \phi} - \tan \phi,$$

which is satisfied when $\omega = 90° - \phi$.

For the case where $\epsilon = 0$,

$$\tan(\phi + \omega) = \frac{\sin \omega}{\cos \omega \tan^2\left(45° - \frac{\phi}{2}\right)},$$

which is satisfied when $\omega = 45° - \frac{\phi}{2}$.

Reliability of the Preceding Theory.—The preceding theory is based upon the assumptions that the earth is a homogeneous mass and without cohesion, and the formulas are deduced under the assumption that the surface of the earth is a plane.

All writers on the subject have considered the earth as a homogeneous mass and, with a few exceptions, without cohesion.

Old and recent experiments indicate that cohesion has very little effect upon the pressure of the earth, which explains why it has not been considered by most writers.

The assumption of a plane earth-surface is necessary whenever practical formulas and direct graphical constructions for obtaining the thrust of the earth are obtained. General formulas can be deduced for any character of surface, but they are too complex for practical use. Those graphical constructions which do not require a plane earth-

surface are not direct in their solution of the problem, but require a series of trials to obtain the maximum thrust.

If the earth-surface is not a plane, one can be assumed which will give the thrust of the earth sufficiently exact for all practical purposes.

For unconfined earth no exceptions can be taken to the preceding theory, the assumptions upon which it is based being accepted, and for confined earth the theory must be true when the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

For all cases in which α and ϵ are positive the theories of *Rankine*, *Winkler*, *Weyrauch*, and *Mohr* agree and give identical results with the preceding theory, as they should, being founded upon the same assumptions.

When α is negative *Weyrauch* does not consider his theory reliable, and his equations lead to indeterminate results.

Winkler and *Mohr* consider their theories reliable whenever the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

Rankine's method of considering the case where α is negative is equivalent to assuming that the introduction of a wall does not affect the stresses within the mass.

It may be concluded that the preceding theory is perfectly exact when α and ϵ are positive; and when α or ϵ is negative that the stresses obtained will be the maximum which under any circumstances can exist.

For the case where ϵ is negative the stress obtained will be considerably larger than the actual stress (when a wall is introduced), depending upon the magnitude of ϵ. For small values of ϵ the results will be practically correct. For large values of ϵ the following method can be employed in determining the thrust of the earth. The

method depends upon the *assumption* that the pressure of the earth is normal to the back of the wall. This may or may not be the case, but it appears to be the most consistent assumption to make for this rare and not important case.

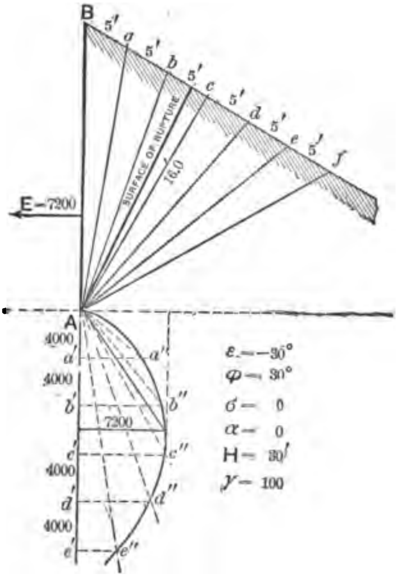


FIG. 9.

* In Fig. 9, let AB be the back of the wall and Bf the surface of the earth. Make $Ba = ab = bc = cd =$ etc. Some prism BAa or BAb or BAc, etc., will produce the maximum thrust on the wall; and when this maximum thrust is produced, the resultant pressure on the plane Aa

* See Van Nostrand's Magazine. XVII, 1877, p. 5. "New Constructions in Graphical Statics," by H. T. Eddy, C.E., Ph.D.

or Ab or Ac, etc., will make the angle ϕ with the normal to the plane.

On the vertical line Ad' lay off $Aa'=a'b'=b'c'$, etc., and draw Aa'' making the angle ϕ with the normal to Aa, Ab'' making the angle ϕ with the normal to Ab, etc.; then draw $a'a''$, $b'b''$, etc., perpendicular to AB, and draw a curve through Aa'', b'', c'', etc. Then there will be a maximum distance parallel to $a'a''$ between Ad' and this curve which will be proportional to the thrust of the earth against AB. This maximum distance multiplied by the altitude $Ac \div 2$ and the product by γ, the weight of a cubic foot of earth, will be the pressure of the earth.

This method is perfectly general and can be applied in any case.

If the earth-pressure is assumed to have the direction given by the formulas of the preceding theory, the construction will give the same value of E, the pressure of the earth.

Some writers assume that the direction of E makes the angle $\phi''=\phi$ with the normal to the back of the wall in all cases. This assumption cannot be correct until the wall commences to tip forward, and then it is doubtful that such is the case unless the earth and wall are perfectly dry.

To be on the side of safety in every case, it is better to take the direction of E as given by the above theory.

The construction of Fig. 9 will give the maximum thrust for any assumed direction for any case.

FORMULAS FOR EARTH-PRESSURE.

IN the following formulas α and ϵ are considered as *positive,* and the wall is assumed to be one foot long.

CASE I. *General case of inclined earth-surface and inclined back of wall.*

$$E = \frac{H^2\gamma}{2}\frac{\cos(\epsilon - \alpha)}{\cos^2\alpha\cos\epsilon} \times$$

$$\sqrt{\begin{array}{l}\sin^2\alpha + \cos^2(\epsilon - \alpha)\left\{\dfrac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}}\right\}^2 \\ + 2\sin\epsilon\sin\alpha\cos(\epsilon - \alpha)\left\{\dfrac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}}\right\}\end{array}}; \quad (1)$$

or

$$E = \frac{H^2\gamma}{2}(B)\sqrt{(C) + (D)A^2 + (E)A}. \quad . \quad (1')$$

$$\tan\delta = \frac{\sin\alpha\cos\epsilon + \sin\epsilon\cos(\epsilon - \alpha)A}{\cos\epsilon\cos(\epsilon - \alpha)A}; \quad . \quad (1a)$$

or $$\tan\delta = \frac{\sin\alpha}{\cos(\epsilon - \alpha)A} + \tan\epsilon, \quad . \quad . \quad . \quad . \quad (1'a)$$

where

$$A = \cos\epsilon \frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}}. \quad . \quad . \qquad (d)$$

CASE II. *Surface of earth inclined and* $\alpha = 0$.

$$E = P = \frac{H^2\gamma}{2}\left\{\cos\epsilon \frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}} = A\right\}. \qquad (2)$$

From Diagram I the values of A can be found for all values of ϕ from 0° to 90° and of ϵ from 0° to 90°, varying by 5°.

$$\delta = \epsilon; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2a)$$

or for all vertical walls the direction of the earth-pressure is parallel to the surface of the earth.

CASE III. *The surface of the earth parallel to the surface of repose.*

$$\epsilon = \phi.$$

$$E = \frac{H^2\gamma}{2} \frac{\cos(\phi - \alpha)}{\cos^2\alpha\cos\phi} \sqrt{\frac{\sin^2\alpha + \cos^2(\phi - \alpha)}{+ 2\sin\alpha\sin\phi\cos(\phi - \alpha)}}. \qquad (3)$$

$$\tan\delta = \frac{\sin\alpha + \sin\phi\cos(\phi - \alpha)}{\cos\phi\cos(\phi - \alpha)}. \quad . \quad . \quad . \qquad (3a)$$

CASE IV. *The surface of the earth parallel to the surface of repose and the back of the wall vertical.*

$$\epsilon = \phi \quad \text{and} \quad \alpha = 0.$$

$$E = \frac{H^2\gamma}{2}\cos\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

$$\delta = \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4a)$$

CASE V. *The surface of the earth horizontal.*

$$\epsilon = 0.$$

$$E = \frac{H^2\gamma}{2}\sqrt{\tan^2\alpha + \tan^4\left(45^\circ - \frac{\phi}{2}\right)}. \quad . \quad (5)$$

$$\tan\delta = \frac{\tan\alpha}{\tan^2\left(45^\circ - \frac{\phi}{2}\right)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

CASE VI. *The surface of the earth horizontal and the back of the wall vertical.*

$$\epsilon = 0 \quad \text{and} \quad \alpha = 0.$$

$$E = \frac{H^2\gamma}{2}\tan^2\left(45^\circ - \frac{\phi}{2}\right) \quad . \quad . \quad . \quad . \quad (6)$$

$$\delta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

CASE VII. *Fluid pressure.*

$$\epsilon = \phi = 0.$$

$$E = \frac{H^2\gamma}{2\cos\alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\delta = \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

GRAPHICAL CONSTRUCTIONS FOR DETERMINING THE THRUST OF EARTH.

The following constructions are perfectly general, and apply to *any plane* within a mass of earth. When applied

for determining the thrust of earth against a ***retaining-wall,*** α and ϵ are taken as *positive.*

* *Construction* (a).

Let BE represent the surface of the earth and BA the back of the wall. Draw AF parallel to BE, and at any point D in AF lay off DF equal to the vertical DE. Draw

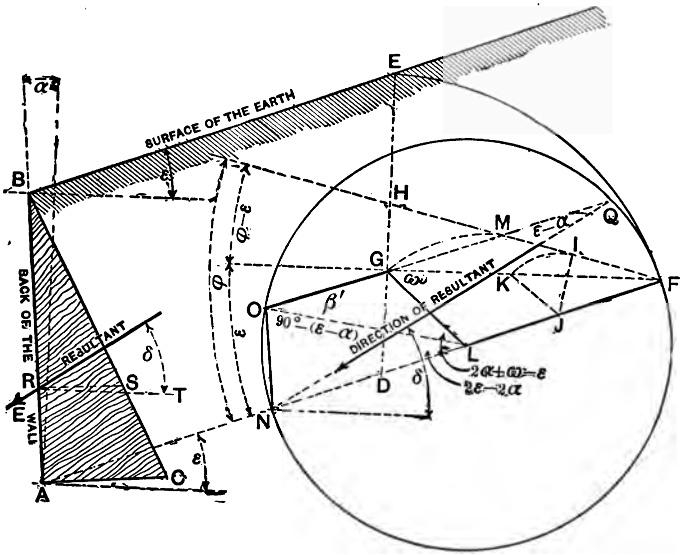


FIG. 10.

FG horizontal, and FH, making the angle ϕ with DF. With any point J in DF describe the arc KI tangent to HF at I cutting FG at K, and draw GL parallel to KJ; with L as a centre and LF as radius, describe the circumference $FQON$ cutting AD at N. Through N draw NO

* See "Theorie des Erddruckes auf Grund der neueren Anschauungen," by Prof. Weyrauch, 1881.

parallel to AB cutting the circumference $FQON$ at O; at A draw AC equal to OG and normal to AB; the area of the triangle ABC multiplied by γ will be the thrust of the earth on the wall.

To determine the direction of the thrust E, prolong OG to Q; then QN will be the direction of the thrust.

This thrust acts on the wall at $\frac{2}{3}AB$ below B.

* *Construction* (*b*).

Let BQ represent the surface of the earth, and BA the back of the wall. Draw AD parallel to BQ, and at any

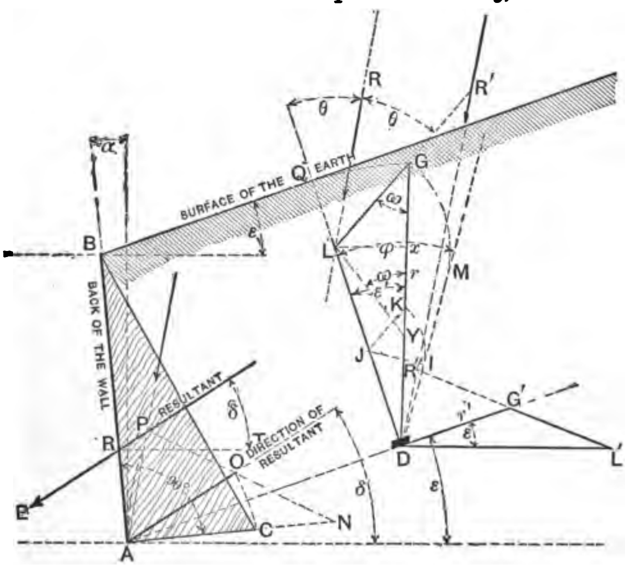


FIG. 11.

point D in AD draw the vertical DG equal to the normal DQ; draw DM making the angle ϕ with the normal DQ.

* This construction follows directly from Rankine's Ellipse of Stress. See Rankine's Applied Mechanics.

At any point J in DQ as a centre, describe the arc IK tangent to DM cutting DG at K, and draw GL parallel to JK. Bisect the angle QLG, and at A draw AP parallel to LR. At A draw AN normal to AB and equal to DL; with N as a centre and AN as radius, describe an arc AP cutting AP at P; connect P and N, and make NO equal to LG; with A as a centre and AO as a radius, describe the arc OC cutting AN at C; then the area of the triangle ABC multiplied by γ will be the thrust against the wall. The direction of this thrust is parallel to AO and it is applied at $\frac{2}{3}AB$ below B.

The constructions (a) and (b) give identical results in every case.

STABILITY OF TRAPEZOIDAL WALLS

As the majority of walls retaining earth are trapezoidal in section, the stability of such walls alone will be considered. If other forms occur in practice they can be divided into trapezoidal sections with horizontal beds, and the stability of each considered, commencing with the upper section.

Walls having the rear faces in the form of steps can usually be considered as trapezoidal in section by replacing the stepped portion by a straight line which approximately bisects each step. If the front faces are stepped they can be treated in a similar manner.

In case the front face of the wall is curved in profile, the curve may be replaced by straight lines which are chords of the curve, thus binding the section into as many trapezoids as there are chords.

It will be assumed that the direction and magnitude of the earth-pressure is known, that the position and extent of the back of the wall, and the width of the top are given,

to determine the width of the base for stability against overturning, sliding, and crushing of the material.

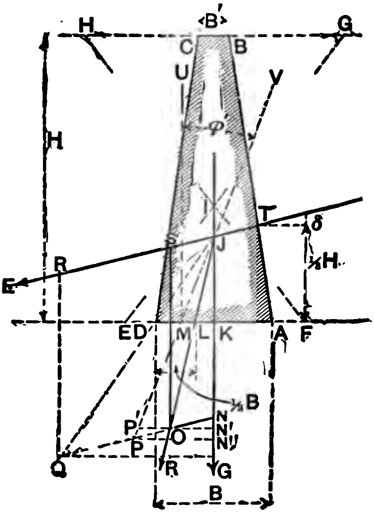


Fig. 12.

Stability against Overturning.—Let $ABCD$, Fig. 12, represent a section of a trapezoidal wall, TR the direction of the earth-thrust, JG the vertical passing through the centre of gravity of the wall, and JO the direction of the resultant pressure on the base AD caused by E and G.

As long as R cuts the base AD, the wall will be stable against overturning. When R takes the direction JQ, the wall may be said to be on the point of overturning; then the factor of safety against overturning is $\frac{QN}{ON}$, where ON is the actual value of E, and QN the value of E required to make the resultant R pass through D.

Stability against Sliding.—Since the wall will not slide

along the surface DA until the resultant R makes an angle with the normal to DA greater than the angle of friction ϕ', the factor of safety against sliding can be obtained as follows: Draw JP making the angle $JMU = \phi'$; then the factor of safety against sliding is $\frac{PN}{ON}$, where PN is the force required in the direction of E to make R make the angle ϕ' with the normal to AD, and ON the actual value of E.

Stability against the Crushing of the Material.—In ordinary practice walls for retaining earth are not of sufficient height to cause very large pressures at their bases, but it is necessary to consider the subject on account of the tendency of the bed-joints to open under certain conditions.

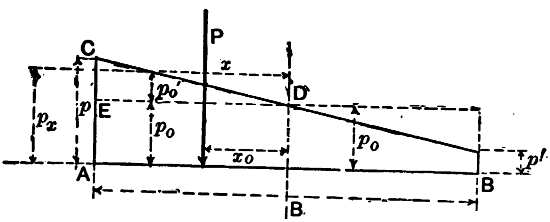


FIG. 13.

Let AB, Fig. 13, represent any bed-joint in the wall, P the vertical resultant pressure upon the joint, and x_0 the distance of the point of application from the centre of the joint.

The intensity of P at any point can be considered as composed of a uniform intensity $p_0 = \frac{P}{B}$, and a uniformly varying intensity p_0', so that $p_x = p_0 + p_0'$. Let a equal the tangent of the angle CDE, then $p_0' = ax$ and $p_x = p_0 + ax$.

The pressure upon a surface (dx)—the joint being considered unity in the dimension normal to the page—is

$$p_x dx = p_0 dx + axdx,$$

and the moment of this about DB is

$$(p_0 dx + axdx)x.$$

The algebraic sum of these moments for values of x between the limits $\pm \frac{B}{2}$ must equal Px_0, or

$$Px_0 = \int_{-\frac{1}{2}B}^{+\frac{1}{2}B} (p_0 xdx + ax^2 dx).$$

Integrating,

$$a = \frac{12x_0 P}{B^3} = \frac{12x_0 p_0}{B^2},$$

and

$$p_x = \frac{B^2 + 12xx_0}{B^2} p_0,$$

or making $x = \frac{1}{2}B$,

$$p = \left\{1 + \frac{6x_0}{B}\right\} \frac{P}{B};$$

and if x_0 be replaced by $\frac{1}{2}B - Q$, where Q is the distance from A to the point where P cuts the base, (Fig. 13,)

$$p = 2\left(2 - \frac{3Q}{B}\right) \frac{P}{B},$$

and

$$p' = 2\left(-1 + \frac{3Q}{B}\right)\frac{P}{B}.$$

If $Q = \frac{1}{3}B$,

$$p' = 0 \quad \text{and} \quad p = 2p_0,$$

from which it is seen that when R cuts the base outside the middle third, the joint will have a tendency to open at points which are at a maximum distance from R where it cuts the base.

Therefore in no case should the resultant pressure be permitted to cut the base outside the middle third. This makes it unnecessary to consider the stability against overturning.

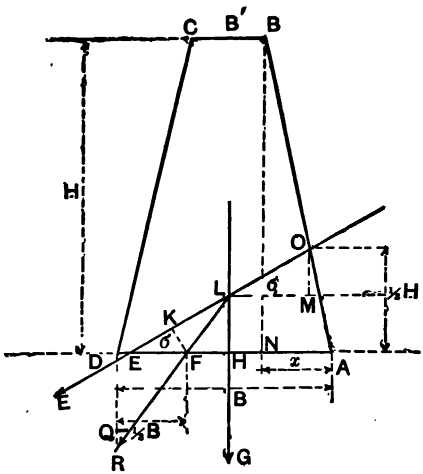


FIG. 14.

Then in designing a wall the following conditions must exist for stability:

I. *The resultant R must cut the base for stability against overturning.*

II. *The resultant R must not make an angle with the normal to the base of the wall greater than the angle of friction* ϕ'.

III. *The resultant R must not cut the base outside of the middle third, in order that there may be no tendency for the bed-joints to open.*

The above three conditions apply to any bed-joint of the wall; but if they are satisfied at the base and the wall has the section shown in Fig. 14, it will not be necessary to consider any joints above the base unless the character of the stone or the bonding is different.

Determination of the width of the base of a retaining-wall under the condition that R cuts the base at a point $\frac{1}{3}B$ from the toe of the wall.

Let H, B', x, δ, and E be given to determine B.

From Fig. 14,

$$KF = \frac{x}{3}\sin\delta + \frac{H}{3}\cos\delta - \frac{2B}{3}\sin\delta,$$

$$HD = \frac{2B^2 + 2BB' - Bx - 2B'x - B'^2}{3(B+B')},$$

$$HF = HD - \frac{B}{3} = \frac{B^2 + BB' - Bx - 2B'x - B'^2}{3(B+B')}.$$

For equilibrium

$$E(KF) = G(HF) = \frac{B+B'}{2}HW(HF).$$

Substituting the values of KF and HF in the above and reducing, it becomes

$$B^2 + B\left(\frac{4E}{HW}\sin\delta + B' - x\right)$$
$$= \frac{2E}{HW}(H\cos\delta + x\sin\delta) + 2B'x + B'^2, \quad . \quad (8)$$

which is the general equation for the width of the base of a trapezoidal wall.

For a rectangular wall $B' = B$.

For a triangular wall $B' = 0$.

For a wall with a vertical front $B' + x = B$ or $B' = B - x$.

For a wall with a vertical back $x = 0$.

Equation (8) is easily transformed to satisfy the requirements of special cases.

The width of the base can be found graphically by assuming a value for B and finding the value of Q; if it is less than $\frac{1}{3}B$ another value of B must be assumed, and so on until Q is equal to or greater than $\frac{1}{3}B$.

Formulas for Trapezoidal and Triangular Walls.

Formulas for the width of the base of trapezoidal walls under the condition that the resultant R cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or $Q = \frac{1}{3}B$.

Case I. *The general case in which the back of the wall is inclined, and E makes an angle with the horizontal.*

$$B^2 + B\left(\frac{4E}{HW}\sin\delta + B' - x\right)$$

$$= \frac{2E}{HW}\left(H\cos\delta + x\sin\delta\right) + 2B'x + B'^2. \quad . \quad (8)$$

Case II. *The back of the wall vertical.*

$$x = 0.$$

$$B^2 + B\left(\frac{4E}{HW}\sin\delta + B'\right) = \frac{2E}{W}\cos\delta + B'^2. \qquad (9)$$

CASE III. *The back of the wall vertical and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B^2 + BB' = \frac{2E}{W} + B'^2. \quad . \quad . \quad . \quad . \quad (10)$$

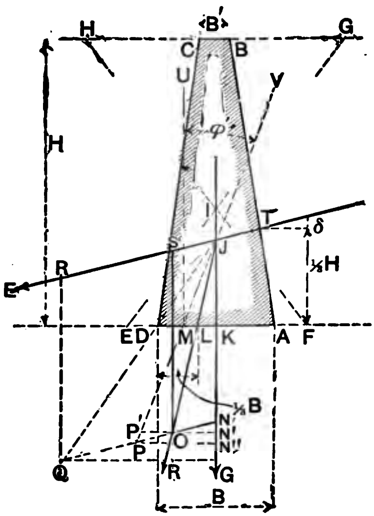


FIG. 15.

If $B = B'$ and $x = 0$, the section of the wall is a rectangle, and (9) becomes

$$B^2 + B\frac{4E}{HW}\sin\delta = \frac{2E}{W}\cos\delta, \quad . \quad . \quad . \quad (9a)$$

and (10) becomes

$$B = \sqrt{\frac{2E}{W}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

Formulas for the width of the base of triangular walls under the condition that the resultant R cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or $Q = \frac{1}{3}B$.

CASE I. *The general case in which the back of the wall is inclined, and E makes an angle with the horizontal.*

$$B^2 + B\left(\frac{4E}{HW}\sin\delta - x\right) = \frac{2E}{HW}(H\cos\delta + x\sin\delta). \quad (11)$$

CASE II. *The back of the wall vertical.*

$$\alpha = 0.$$

$$B^2 + B\left(\frac{4E}{HW}\sin\delta\right) = \frac{2E}{W}\cos\delta. \quad . \quad . \quad (12)$$

CASE III. *The back of the wall vertical, and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B = \sqrt{\frac{2E}{W}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The above formulas do not contain the condition that R shall not make an angle greater than ϕ' with the normal to the base of the wall.

From Fig. 15,

$$\tan\phi' \geqq \frac{E\cos\delta}{G + E\sin\delta} = \tan LJK, \quad . \quad . \quad (14)$$

which expresses the condition under which the wall will not slide.

FOUNDATIONS FOR WALLS RETAINING EARTH.

The design of the foundations for retaining-walls has received but little attention by writers upon engineering subjects, and the practical engineer has not published to any great extent examples of the foundations he has employed under the countless number of walls erected along railways, highways, canals, etc.

As the designing of foundations resting upon earth, for walls retaining earth, introduces several features which do not influence the ordinary cases of foundations, it will be best to make a special investigation for such conditions.

The intensity of the foundation pressure upon the earth is seldom uniform, due principally to the pressure of the earth behind the wall and foundation tending to overturn the structure as a whole; this being the case, evidently the maximum intensity upon the earth must not be large enough to heave the earth, and the minimum intensity must not be so small that the earth may heave the foundation.

If the foundation be so designed that neither it nor the earth can be heaved, the structure may yet fail by sliding forward. This can only be resisted by the abutting power of the earth in front of the foundation and the friction upon the base of the foundation. Usually, however, if there is no danger of any movement in a vertical plane, there is little or no danger of any movement in a horizontal direction.

As in any structure good judgment must enter into the design, the formulas which will be demonstrated must be

used as guides only. These formulas will depend upon the angle of repose ϕ of a homogeneous granular mass, and the specific gravity of this mass. For ordinary earths for which the weights and angles of repose are known the results obtained by the use of the formulas will compare very favorably with those obtained from examples of the best practice.

Depth of Foundations.—Given the angle of repose ϕ of any earth, to determine the depth to which it is necessary to sink a foundation to support a given load. The surface of the earth is assumed to be horizontal.

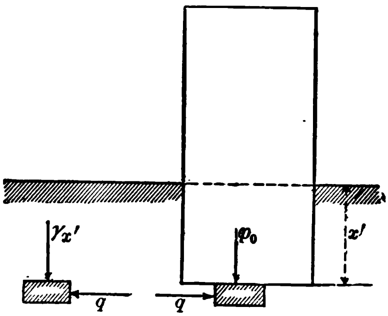


FIG. 16.

CASE I. *When the intensity of the pressure on the base of the foundation is uniform.*

In Fig. 16, let p_0 represent the intensity of the pressure on the base of the foundation.

Now when the masonry is about to sink (see Eq. (c)),

$$\frac{p_0}{q} = \frac{1 + \sin \phi}{1 - \sin \phi} \quad \text{or} \quad q = p_0 \frac{1 - \sin \phi}{1 + \sin \phi}.$$

If x' represents the depth to which the foundation extends below the surface of the earth and γ the weight of a cubic

foot of earth, then $\gamma x'$ equals the vertical intensity of the earth-pressure on a plane at the depth of the lowest point of the foundation.

When the wall is on the point of sinking, the earth must be on the point of rising, or

$$\frac{q}{\gamma x'} = \frac{1 + \sin\phi}{1 - \sin\phi},$$

or

$$p_0 = \gamma x' \left\{ \frac{1 + \sin\phi}{1 - \sin\phi} \right\}^2. \quad . \quad . \quad . \quad . \quad (15)$$

In any case p_0 must not have a greater value than that obtained from (15)—

$$x' = \frac{p_0}{\gamma} \left\{ \frac{1 - \sin\phi}{1 + \sin\phi} \right\}^2 = \frac{p_0}{\gamma} \tan^4 \left(45^\circ - \frac{\phi}{2}\right). \qquad (16)$$

The value of x' as obtained from (16) is the least allowable value consistent with equilibrium. Since x' is a function of $\tan^4\left(45^\circ - \frac{\phi}{2}\right)$, care must be taken that ϕ is assumed at its least value. As ϕ becomes smaller the value of x' increases rapidly.

CASE II. *When the intensity of the pressure on the base is uniformly varying.*

Let p represent the maximum intensity of the pressure on the earth and p' the minimum intensity; then for equilibrium p must not exceed the value obtained from the following equation (see 15):

$$p = x'\gamma \left\{ \frac{1 + \sin\phi}{1 - \sin\phi} \right\}^2. \quad . \quad . \quad . \quad . \quad (17)$$

For any assumed depth x' the maximum value of p can be

found from (17). For any assumed breadth B'' of the foundation the value of p due to the resultant pressure upon the base of the foundation can be found from the formulas on page 31, when the value of x_0 has been determined; this value must not be greater than the value of p found from (17), or the masonry will heave the earth.

In order that the earth may not heave the masonry, p' must not be *less* than the value obtained from the following formula:

$$p' = x'\gamma \left\{ \frac{1 - \sin\phi}{1 + \sin\phi} \right\}^2 . \quad . \quad . \quad . \quad . \quad (18)$$

Then

$$p_0 = \frac{p + p'}{2} = \frac{x'\gamma}{2} \left\{ \left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2 + \left(\frac{1 - \sin\phi}{1 + \sin\phi}\right)^2 \right\}, \quad (19)$$

which expresses the *maximum* value p_0 can have for the equilibrium of the earth and the masonry.

In order that p' may never be less than the value obtained from (18), the resultant pressure upon the base of the foundation must cut the base within a certain distance of its centre. If x_0 be this distance, then (page 31)

$$p' = x'\gamma \left\{ \frac{1 - \sin\phi}{1 + \sin\phi} \right\}^2 = \left\{ 1 - \frac{6x_0}{B''} \right\} p_0. \quad (20)$$

Substituting the value of p_0 from (19) and solving for x_0,

$$x_0 = \frac{B''}{6} \left\{ \frac{X - Y}{X + Y} \right\}, \quad . \quad . \quad . \quad . \quad (21)$$

where

$$*X = \left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2 \quad \text{and} \quad Y = \left(\frac{1 - \sin\phi}{1 + \sin\phi}\right)^2.$$

---

* Tabulated values of X and Y are given on page 72.

Depth of foundations when the surface of the earth has different elevations on opposite sides of the structure.

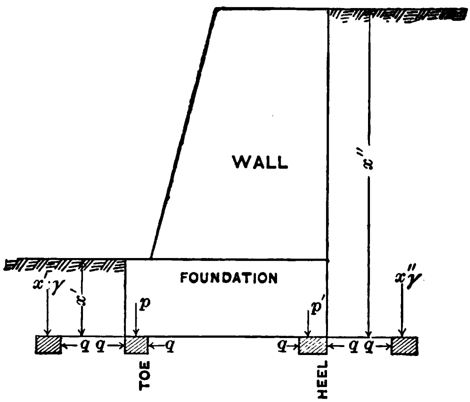


FIG. 17.

This case is illustrated in Fig. 17. From (17) and (18) for equilibrium

$$p \leqq x'\gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and

$$p' \geqq x''\gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^2 \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Combining (22) and (23) in the value of p_0,

$$p_0 = \frac{p + p'}{2} = \frac{\gamma}{2} \left\{ x' \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^2 + x'' \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 \right\} . \quad (24)$$

Having assumed the values of γ and ϕ for any particular case, the above formulas determine the permissible magni-

tudes of the intensities at the heel and toe of the foundation for any depth. The breadth of the base of the foundation may now be assumed, and the actual intensities compared with those permissible; if p is too large or p' too small, another trial must be made. Usually one or two trials are sufficient. If one prefers to compute the width of the base of a trapezoidal foundation, the formula given below can be employed.

Determination of the breadth B'' of a trapezoidal foundation for a given loading and a maximum intensity p at the toe. (Back of foundation vertical.)

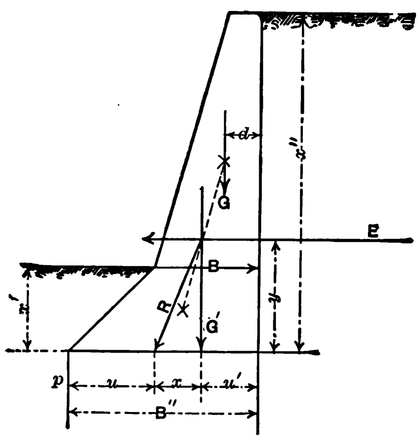


FIG. 18.

Let G = total vertical weight supported by top of foundation;

E = thrust of earth;

p = maximum intensity of pressure at toe of foundation as found from (22);

and B'' = breadth of base of foundation.

Then

$$B'' = \frac{-G + \sqrt{p(6dG + x'B^2W + 6Ey) + G^2}}{p}. \quad (25)$$

The foundation can nearly always be designed as a trapezoid having a vertical back, and then if necessary the batter in front can be stepped. For walls under twenty feet in height, retaining material which will assume a slope of $1\frac{1}{2}$ to 1, the most economical foundation is rectangular in section if the base must be four feet deep to escape the action of frost. Where frost need not be considered, of course more shallow and broader foundations can be employed.

Abutting Power of Earth.—Let the surface of the earth be horizontal and the body pushing the earth have a vertical face; then at the depth x' the maximum horizontal pressure per unit of area is (see Case I above)

$$q = x'\gamma \frac{1 + \sin\phi}{1 - \sin\phi},$$

and since q varies directly as x', the total thrust P which the earth is capable of resisting is

$$P = \frac{(x')^2\gamma}{2}\frac{1 + \sin\phi}{1 - \sin\phi}. \quad . \quad . \quad . \quad (26)$$

Bearing Power of Earth.—The bearing power or the intensity of the pressure which earth can resist depends not only upon the character of the earth, but upon the depth to which the foundation is extended, as shown by the formulas for p given above. For example, the foundation may be very broad and shallow or quite narrow and deep. The

intensity of the pressure in the first case being considerably smaller than in the second, and both conditions fulfilling the conditions of stability. It appears then that the bearing powers of earth given by various writers must be employed with caution, unless the conditions upon which the values were based are known.

APPLICATIONS.

The determination of the earth-pressure by the preceding formulas and graphical constructions is a very simple operation when the angle ϕ has been determined or assumed. That care and judgment be used in assuming the value of ϕ is very important, since a change of a few degrees in the value of ϕ sometimes causes a large change in the value of E. An inspection of Diagram I shows that the value of the coefficient A increases very rapidly as ϕ decreases.

When the earth to be retained contains springs, the bank must be thoroughly drained if it is to be retained by an economical tight wall; if it is not drained, the angle ϕ will be likely to become very small as the earth becomes wet.

When the location of the earth to be retained is subjected to jars, the value of ϕ will be decreased.

Hence, in assuming the value of ϕ, the engineer must be sure that the value assumed will be the least value which, in his judgment, it is likely to have.

In constructing the wall the judgment and authority of the engineer must again be exercised in order that the wall be constructed as designed.

In all cases, to insure perfect drainage between the back

of the wall and the earth, numerous "weep-holes" should be provided in the body of the wall, or proper arrangements made to carry away the water at the base of the wall. To facilitate drainage, the backing resting against the wall should be sand or gravel.

In no case should water be permitted to get under the foundation of the wall, neither should the earth in front of the wall be allowed to become wet.

In cold localities the back of the wall near the top should have a large batter to prevent the frost from moving the top courses of stone. As a guard against sliding, the courses of the wall should have very rough beds. The strength of a wall is increased the nearer it approaches a monolith.

Care should be taken to have the foundation broad and deep enough to prevent sliding and upheaving of the earth in front. In clay the foundation should be deep, while in sand or gravel it may be broad and shallow.

The following examples illustrate the application of the formulas:

Ex. 1. Design a trapezoidal wall of sandstone, weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining forward 5°, to retain a bank of sand sloping upward at an angle of 20°.

Data.

$\gamma = 100$ lbs., $W = 150$ lbs.; $\epsilon = 20°$, $\phi = 39°$, $\alpha = 5°$; $H = 30$ ft., $B' = 3$ ft., $x = 2.63$ ft.

1°. *Graphical determination of the values of E and δ.*

The graphical solution of the problem is shown in Fig. 19, where E is found to equal 15,000 pounds. δ lies between 35° and 36°.

2°. *Algebraic determination of E and δ.*

$$E = \frac{H^2\gamma}{2}(B)\sqrt{(C) + (D)A^2 + (E)A}. \quad . \quad . \quad . \quad . \quad . \qquad (1')$$

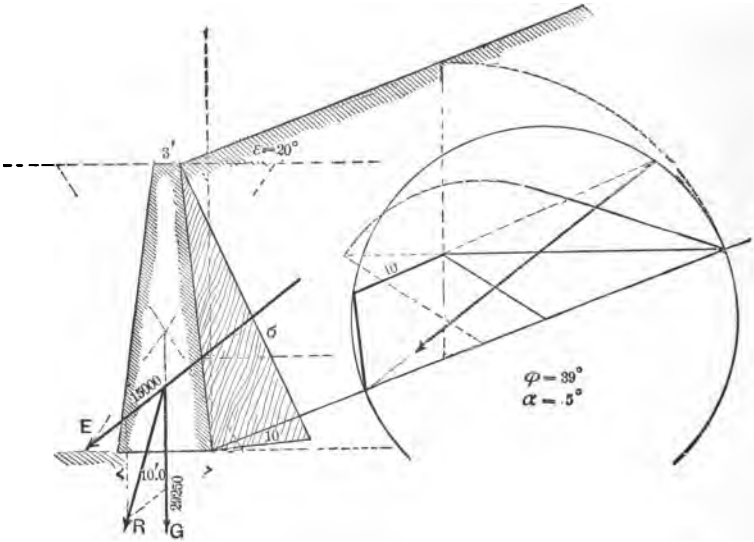


FIG. 19.

Substituting the values of *B, C, D,* and *E* as given in the tables, and that of *A* as given by Diagram I, this becomes

$$E = \frac{900 \times 100}{2}(1.036) \times$$

$$\sqrt{(0.008)+(1.057)(0.264)^2+(0.061)0.264},$$

$$E = 45{,}000\ (1.036)\sqrt{0.098} = 14{,}500 \text{ lbs.}$$

$$\tan\delta = \frac{\sin\alpha}{\cos(\epsilon-\alpha)A} + \tan\epsilon, \quad . \quad . \qquad (1'a)$$

$$\tan\delta = \frac{0.087}{0.966(0.264)} + 0.364,$$

$$\tan\delta = 0.705 = \tan 35^\circ\ 11', \text{ about.}$$

3°. *Algebraic determination of the value of B under the assumption that $Q = \frac{1}{3}B$.*

$$B^2 + B\left\{\frac{4E}{HW}\sin\delta + B' - x\right\}$$
$$= \frac{2E}{HW}\left\{H\cos\delta + x\sin\delta\right\} + 2B'x + B'^2. \quad . \quad (8)$$

$$B^2 + B\left\{\frac{4\times 14500}{30\times 150}\,0.576 + 3 - 2.63\right\}$$
$$= \frac{2\times 14500}{30\times 150}\{30\times 0.817 + 2.63\times 0.576\} + 6\times 2.63 + 9,$$

$$B^2 + 7.79B = 172.53,$$

$$B = -\,3.89 \pm \sqrt{172.53 + \overline{3.9}^2};$$

$$\therefore B = 13.69 - 3.89 = 9.80 \text{ ft.};$$

or, practically, 10 feet is the required width of the base.

4°. *To determine if the wall will slide on a foundation of sandstone.*

From (14),

$$\tan\phi' \geqq \frac{E\cos\delta}{G + E\sin\delta}.$$

Taking $B = 10$ ft., $G = \frac{10+3}{2}30\times 150 = 29250$ lbs.

$\delta = 35° \; 11'$, $\cos \delta = 0.817$, and $\sin \delta = 0.576$, then

$$\frac{E \cos \delta}{G + E \sin \delta} = \frac{14500 \times 0.817}{29250 + 14500 \times 0.576} = 0.315.$$

From Table II, the value of $\tan \phi'$ for masonry is 0.6 to 0.7; hence there is no danger of the wall sliding on the foundation.

According to the *Engineering News* formula the base of this wall would be $\frac{3}{7}H$ "plus a few inches for good luck," or about 13 feet, and by the old rule of one third the height 10 feet.

Ex. 2. Design a trapezoidal wall of sandstone weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining backward 15°, to retain a bank of sand sloping upward at an angle of 30°.

Data.

$\gamma = 100$ lbs., $W = 150$ lbs.; $\epsilon = 30°$, $\phi = 33°$, $\alpha = -15°$; $H = 30$ ft., $B' = 3$ ft., $x = 8$ ft.

1°. *Graphical determination of the values of E and δ.*

In Fig. 19, let BG represent the surface of the earth, and AB the back of the wall. Draw AF parallel to BG, and from any point D' in AF lay off $D'F$ equal to the vertical $D'G$, and draw FL horizontal; lay off the angle $IFD' = \phi = 33°$, and locate the point M in $D'F$ so that if an arc be described with M as a centre and LM as a radius the arc will be tangent to IF; then with M as a centre and MF as a radius, describe the circumference FHJ and draw JH

parallel to AB; at A draw AL perpendicular to AB and equal to HI. Then

$$\frac{(AB)(AL)}{2}\gamma = \frac{(30.9)(9.6)}{2}100 = 14800 = E.$$

To determine δ, prolong HI to K and draw KJ. Then the angle which this line makes with the horizontal is equal to δ, which is 6° to 7° in this case.

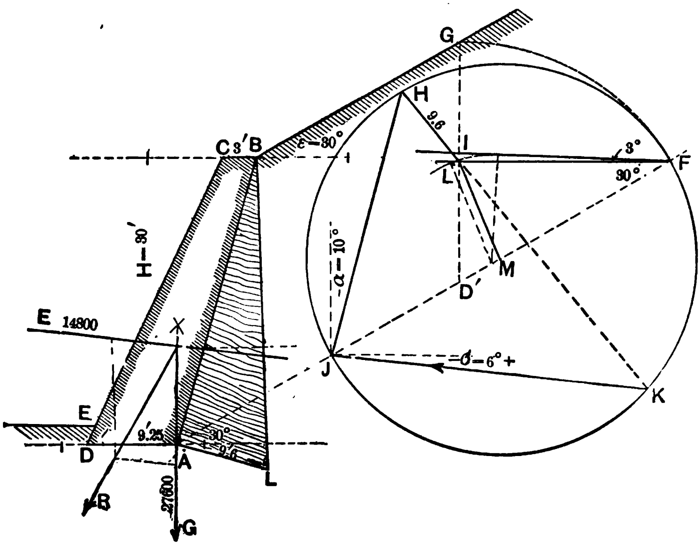


FIG. 20.

2°. *Algebraic determination of E and δ.*

Substituting in (1) and remembering that α is negative,

$$E = 45000\,(0.875)\,\sqrt{0.067 + 0.183 - 0.111} = 14600 \text{ lbs.}$$

From (1'*a*),

$$\tan \delta = \frac{-0.259}{0.707(0.524)} + .577 = -0.123 = \tan(-7°).$$

3°. *Algebraic determination of the value of B under the assumption that* $Q = \frac{1}{3}B$.

Substituting the proper values in (11) and remembering that α is negative,

$$B = -4.7 \pm \sqrt{163.44 + (4.7)^2} = 9.0 \text{ ft.}$$

Ex. 3. Determine the dimensions of a brick wall having a vertical back to retain a bank of sand sloping upward at an angle of 20°. $\phi = 30°$, $H = 20'$, $B' = 2'$, $\gamma = 100$.

1°. *Algebraic determination of E and* δ.

Since $\alpha = 0$,

$$E = \frac{H^2\gamma}{2}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$E = \frac{400 \times 100}{2} 0.424 = 8480; \text{ say, } 8500 \text{ lbs.}$$

The value of A is readily found from Diagram I.

$$\delta = \epsilon = 20°, \quad \text{since} \quad \alpha = 0.$$

2. *Algebraic determination of the value of B under the condition that* $Q = \frac{1}{3}B$.

$$B^2 + B\left\{\frac{4E}{HW}\sin\delta + B'\right\} = \frac{2E}{W}\cos\delta + B'^2. \quad (9)$$

From Table I, $W = 125$ lbs. Then

$$B^2 + B\left\{\frac{4 \times 8500}{20 \times 125}0.342 + 2\right\} = \frac{2 \times 8500}{125}0.940 + 4,$$

or

$$B^2 + 6.65B = 131.84.$$

$$B = -3.36 \pm \sqrt{131.84 + \overline{3.36}^2},$$

and

$$B = -3.36 + 11.96 = 7.60 \text{ ft.}$$

Ex. 4. Determine the value of B in Ex. 3 under the assumption that $\epsilon = 0$ (horizontal earth-surface).

$$E = \frac{H^2\gamma}{2}\left\{\tan^2\left(45° - \frac{\phi}{2}\right) = \frac{1 - \sin\phi}{1 + \sin\phi}\right\}, \quad . \quad (6)$$

or $E = 20000\,(0.333) = 6666$, say 6700 lbs.
Since $\alpha = 0$, and $\epsilon = 0$, $\delta = 0$,

$$B^2 + BB' = \frac{2E}{W} + B'^2; \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$B^2 + 2B = 111.2;$$

$$B = -1 \pm \sqrt{111.2 + 1},$$

and

$$B = -1 + 10.59 = 9.6 \text{ ft.}$$

Ex. 5. Determine the value of B in Ex. 3, under the assumption that $\epsilon = \phi = 30°$.

$$E = \frac{H^2\gamma}{2}\cos\phi = 20000\,(0.866) = 17320 \text{ lbs.}$$

From (9),

$$B^2 + B\left\{\frac{4 \times 17320}{20 \times 125}0.5 + 2\right\} = \frac{2 \times 17320}{125}0.866 + 4;$$

$$B^2 + 15.86B = 244.05;$$

$$B = -7.93 + \sqrt{244.05 + \overline{7.93}^2}.$$

and $\quad B = -7.93 + 17.52 = 9.6$ ft.

Ex. 6. Determine the resultant pressure against the back of a wall when the surface of the earth carries a load equivalent to 5 feet in depth of sand.

$H = 30$ ft., $\alpha = 10°$, $\phi = 30°$, $\epsilon = 0$, and $\gamma = 100$ lbs.

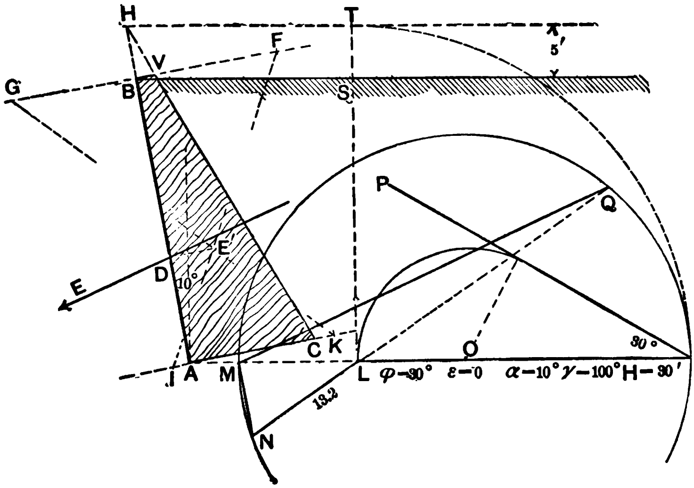


FIG. 21.

Graphical solution of the problem.—In Fig. 21, let BS represent the surface of the earth, and BA the back of the wall.

Make $ST = 5$, and draw HT and BH. Draw AR parallel to BS, parallel to HT, and make LR equal to LT; lay off the angle LRP equal to 30°; with O as a centre

draw an arc passing through L tangent to PR, and then with OR as a radius describe the circumference of the circle RQM, and at M draw MN parallel to AH; at A and normal to AH draw AC equal to NL. Then

$$\frac{AC + BV}{2} BA \cdot \gamma = E.$$

The direction of E will be parallel to QM.

To determine the point of application of E, find the centre of gravity E' of $ABVC$, and draw $E'D$ parallel to AC, then D will be the point of application of E.

E' can be found as follows: Produce AC and BV, make $AI = CK = BV$, $BG = VF = AC$, and join F and I and G and K. Then E', the intersection of FI and GK, will be the centre of gravity of $ABVC$. BD can be found from the formula

$$BD \cos 10^\circ = \frac{1}{3} \frac{(TL)^3 - 3(TL)(TS)^2 + 2(TS)^3}{(TL)^2 - (TS)^2}.$$

Ex. 7. Determine graphically the value of E when $\epsilon = 0$ and $\alpha = 0$, ϕ, γ, and H being given.

In Fig. 22 let BF represent the surface of the earth, and AB the back of the wall. Draw AL parallel to BF and make $IL = IF$; lay off the angle $GLH = \phi$, and at any point K in LH draw MK perpendicular to HL, and lay off $MO = MK$; draw MJ parallel to OI. Then will the arc IN, described with J as a centre and IJ as a radius, pass through I and be tangent to GL; with J as a centre and JL as radius describe the circumference LH; at A lay off $AC = HI$ and normal to AB. Then

$$\frac{AC \times AB}{2} \gamma = E.$$

E is parallel to BF and applied at D, AD being equal to $\frac{1}{3}AB$.

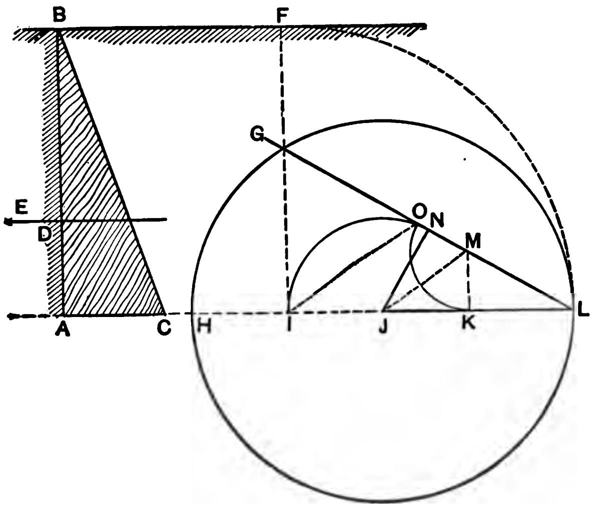


FIG. 22.

Ex. 8. Determine the earth-thrust on the profile shown in Fig. 23, H, γ, ϕ, and ϵ being given.

Graphical solution of the problem.—Let $BCDEA$ represent the given profile, and let the surface of the earth be horizontal. Prolong BC until it intersects SA in S; draw SR normal to BCS and equal to the intensity of the earth-pressure at S; connect B and R. Then from the middle point of BC draw GF parallel to SR; the distance GF multiplied by γ will be the average intensity of the earth-pressure on BC. In a similar manner the average intensities on CD, DE, and EA can be found, and hence the total pressures on each determined. The points of application of these resultant pressures, E_1, E_2, E_3, and E_4,

can be found by the method used in Ex. 6 for finding the centre of gravity of a trapezoid. The directions of

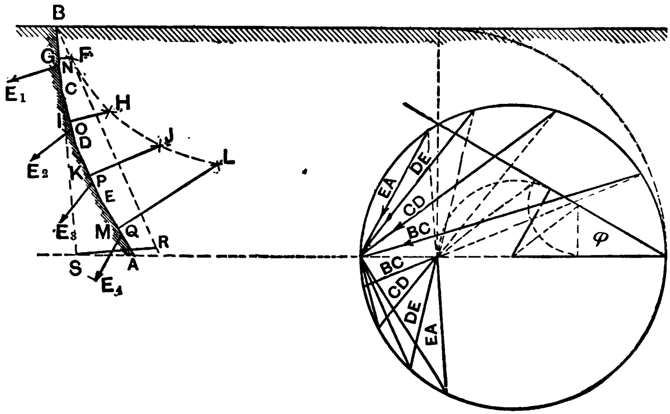


FIG. 23.

E_1, E_2, E_3, and E_4 are found from the construction on the right.

Ex. 9. Determine the thrust of the earth against a vertical wall when ϵ is negative.

For the explanation of this construction, see page 21, Fig. 9.

Ex. 10. From the following data determine E, δ, and Q:

$$\epsilon = 0,\ \phi = 38°,\ \alpha = 10°\ 23';\ \gamma = 90 \text{ lbs.},\ W = 170 \text{ lbs.};$$

$$H = 15 \text{ ft.},\quad B = 6 \text{ ft.},\quad B' = 2 \text{ ft.}$$

Ans. $E = 3037$ lbs., $\delta = 27°\ 13'$, $Q = 2.2$ ft.

Ex. 11. Determine the dimensions of a trapezoidal wall built of dry, rough granite, having a vertical back and being 20 feet high, to safely retain the side of a sand cut,

the surface of the sand being level with the top of the wall. $W = 165$ lbs., $\gamma = 100$ lbs., $\phi = 33° \ 40'$, $H = 20$ ft., $B' = 2$ ft.

Ans. $E = 5734$ lbs., $\delta = 0$, $B = 8$ ft., and $Q = 2.8$ ft., about.

Ex. 12. The same as Ex. 11, with $\alpha = 8°$ instead of $\alpha = 0$.

Ans. $E = 6330$ lbs., $B = 8$ ft., and $Q = 2.7$ ft.

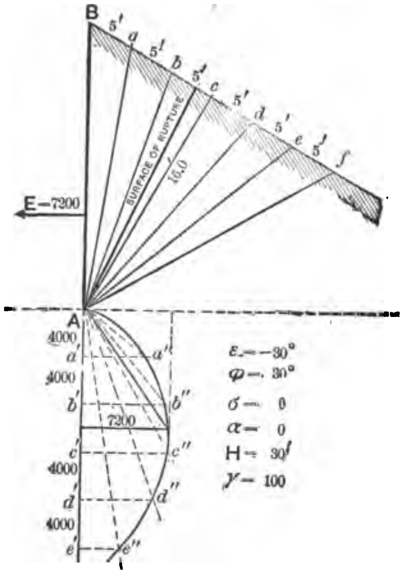


FIG. 24.

Ex. 13. What must be the dimensions of a rubble wall of large blocks of limestone, laid dry, to retain a sand filling which supports two lines of standard-gauge railroad track? (Assume the depth of sand to produce a pressure on the earth equal to that produced by the railroad and trains as 4 feet.)

$H = 15$ ft., $\alpha = 8°$, $\phi = 33° 40'$, $\gamma = 100$ lbs., $W = 170$ lbs., $B' = 3.5$ ft.

Ans. $E = 5760$ lbs., $\delta = 18° 7'$, $B = 8$ ft., $Q = 2.7$ ft.

Ex. 14. Determine E, δ, B, and Q, when $W = 170$ lbs., $\gamma = 100$ lbs., $\alpha = 8°$, $\epsilon = \phi = 33° 40'$, $H = 20$ ft., $B' = 2$ ft.

Ans. $E = 21760$ lbs., $\delta = 32° 25'$, $B = 9$ ft., $Q = 3$ ft.

* Ex. 15. A wall 9 ft. high faces the steepest declivity of earth at a slope of 20° to the horizon; weight of earth 130 lbs. per cubic foot, angle of repose 30°. Determine E when $\alpha = 0$.

Ans. $E = 2187$ lbs.

* Ex. 16. $\epsilon = 33° 42'$, $\phi = 36°$, $H = 3$ ft., $\gamma = 120$ lbs., $\alpha = 0$. Determine E.

Ans. $E = 278$ lbs.

* Ex. 17. $\phi = 25°$, $\epsilon = 0$, $\alpha = 0$, $H = 4$ ft., $\gamma = 120$ lbs., $E = ?$

Ans. $E = 390$ lbs.

* Ex. 18. $\phi = 38°$, $\epsilon = 0$, $\alpha = 0$, $H = 3$ ft., $\gamma = 94$ lbs., $E = ?$

Ans. $E = 100.5$ lbs.

* Ex. 19. A ditch 6 feet deep is cut with vertical faces in clay. These are shored up with boards, a strut being put across from board to board 2 feet from bottom, at intervals of 5 feet apart. The coefficient of friction of the moist clay is 0.287, and its weight 120 lbs. per cubic foot. Find the thrust on a strut, also find the greatest thrust which might be put upon the struts before the adjoining earth would heave up.

Ans. $E = 1230$ lbs.
Thrust per strut = 6128 lbs.
Greatest thrust = 19029 lbs.

---

* Alexander's Applied Mechanics.

Ex. 20. Examine the stability of the wall shown in Fig. 25, and design a foundation which will be safe as long as the condition of the earth remains unchanged; the weight of the masonry being 145 pounds per cubic foot, that of earth 100 pounds, and the angle of repose of the earth such that it will stand at a slope of $1\frac{1}{2}$ to 1.

Stability of the Wall upon the Foundation.—Replacing the stepped back by the line *BD*, the thrust of the earth is found to be about 9900 pounds. The direction of this force is shown in Fig. 25; since it cuts the base of the wall there is no danger of the structure being overturned, however large *E* may become.

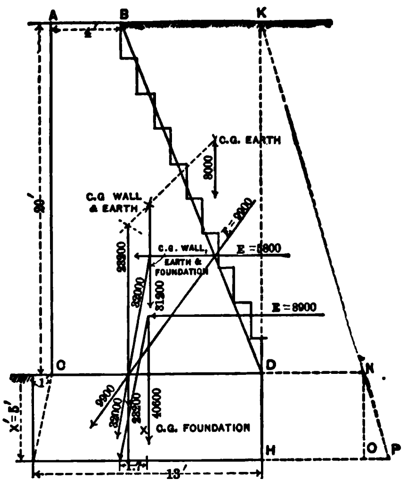


FIG. 25.

Determining the centre of gravity of the wall and also its weight, and combining this with *E*, the resultant pressure upon the base of the wall is found to be about 32,000 pounds. This resultant makes an angle of less than 11

degrees with the normal to the base. Now since for masonry sliding upon masonry the angle of friction is from 31 to 35 degrees (Table II), there is no danger of failure by sliding upon the foundation. Since the resultant cuts the base within the middle third the entire base is subjected to compression, and there will be no tendency for the joints to open at the heel.

Failure by the crushing of the material need not be considered, as the maximum intensity of the pressure upon the base is many times smaller than the ultimate strength of the material. See page 68.

The resultant pressure upon the base can be found also by assuming the earth on the left of the vertical to be supported by the wall, and that the pressure of the earth upon the right of this line acts against the vertical plane KD; this pressure is about 5800 pounds, and is horizontal. Combining this force with the weight of the wall and earth on the left of the line KD, the resultant pressure upon the base is found to be the *same in magnitude and direction as by the first method.*

The Foundation.—The depth of the foundation must be below the action of frost; let this be assumed as 5 feet; then by (22), with $x' = 5$ feet, the *maximum* allowable pressure at the toe of the foundation is about 6000 pounds per square foot, and by (23) the *minimum* allowable pressure is about 200 pounds for $x'' = 25$ feet.

Assuming that the foundation is vertical at the back and trapezoidal in section, the length of the base B'' can be found from (25), which will satisfy the condition of maximum pressure at the toe. Letting $p = 5000$ and $x' = 5$, and solving (25), B'' is found to be between 12 and 13 feet; say 13 feet.

To determine if this width is sufficient to satisfy all the

conditions of equilibrium, the resultant of all forces acting upon the base must be found.

* The total earth-pressure upon the vertical *HK* is about 8900 pounds. Combining this with the weight of the wall, earth supported by the wall, and that of the foundation, the resultant vertical pressure is found to be about 40,600 pounds, and is applied within the middle third of the base, about 1.7 feet to the left of the centre.

The intensity of the pressure at the toe is (page 31)

$$p = \left\{ 1 + \frac{6(1.7)}{13} \right\} \frac{40600}{13} = \text{about 5600 pounds,}$$

which is less than the maximum allowable intensity. The intensity at the heel is $p' = 2p_0 - p =$ about 650 pounds, which is greater than the minimum allowable intensity; hence this foundation is sufficient to prevent settlement.

A glance at Fig. 25 is sufficient to show that the foundation will not slide upon the earth even if the movement were not opposed by a force of some 4000 pounds, being the abutting power of the earth in front of the foundation.

The above foundation then fulfils all the conditions of stability, but to allow for contingencies the foundation should be designed under the assumption that ϕ may be somewhat smaller than its average value, which is equivalent to broadening the base if the depth remains the same

* The pressure against the foundation in front of the wall has been neglected, but can be easily included by taking the area *KHON* instead of *KHP*.

EXAMPLES OF RETAINING-WALL PROFILES.

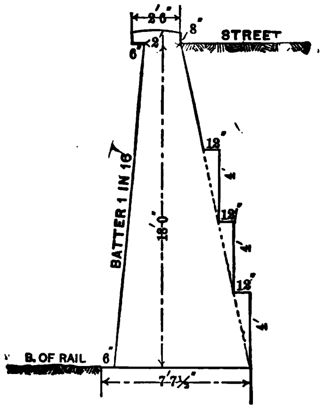


FIG. 26.

A Standard Profile used for the past twenty years near New York City, where railway tracks have been lowered below the streets. (*Engineering News*, 1889.)

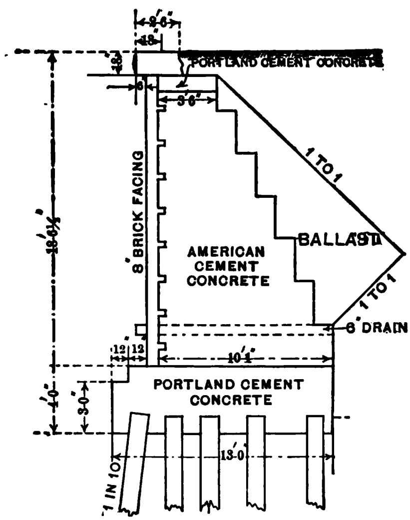


FIG. 27.

Profile of Retaining-wall at Ferdinand Street Bridge, Boston, Mass. (*City Engineer's Report*, 1891.)

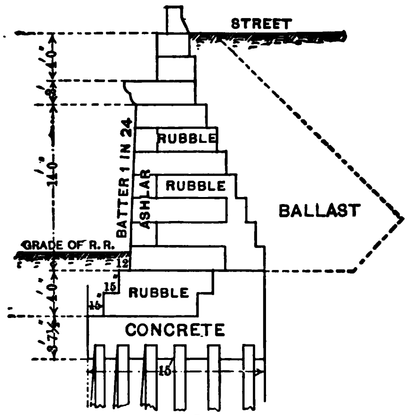


FIG. 28.
Profile of Abutment at Ferdinand Street Bridge, Boston, Mass. (*City Engineer's Report*, 1891.)

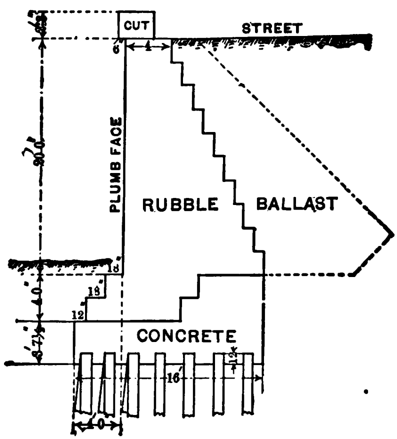


FIG. 29.
Profile of Retaining-wall at Boylston Street Bridge, Boston, Mass. (*City Engineer's Report*, 1888.)

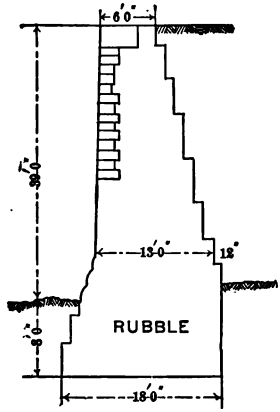


FIG. 30.
Profile of Retaining-wall at Liverpool, England. (*Harcourt*).

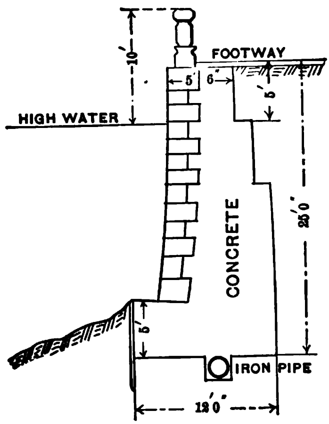


FIG. 31.
Profile of Retaining-wall, Thames Embankment, Chelsea. (*Harcourt.*)

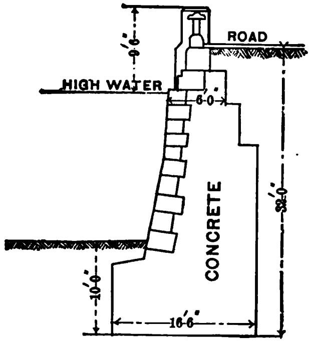


FIG. 32.
Profile of Retaining-wall Thames Embankment, Lambeth. (*Har court.*)

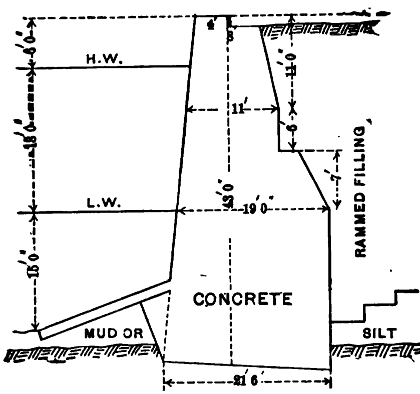


FIG. 33.
Profile of Concrete Retaining-wall at Chatham. (*Harcourt.*)

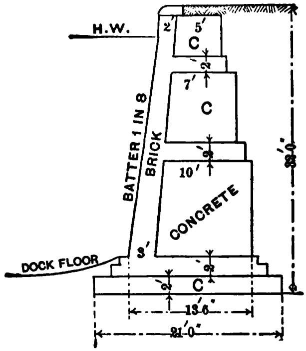


FIG. 34.
Profile of Retaining-wall at Millwall. (*Harcourt.*)

FOUNDATIONS.

The proper proportions of foundations to suit different conditions have been the results of experience principally, though theory enters into their design in many ways. Under certain logical assumptions, the offsets of wood, iron, or stone foundation courses can be as accurately determined as the stresses in any beam subjected to cross-bending. The strengths of various materials which enter into the construction of foundations have been fairly well determined experimentally, so that the allowable intensities of the pressures, and consequently the areas of the foundation courses, can be accurately determined. There remains the most difficult portion to be decided, namely, the proper intensity of the pressure upon the earth which must support the load. Under certain assumptions this can be computed, but the best of judgment must be exercised in making the assumptions upon which calculations are based.

Whenever possible, the intensity of the pressure upon the earth should be uniform under all parts of the structure (assuming the earth to be homogeneous), *and the foundations extend to the same depth.* Theoretically, a greater intensity is allowable at a greater depth, but practically this may lead to unequal settlement, due to the compressibility of the earth, which theory does not take into account.

FOUNDATIONS UPON ROCK.

In preparing a bed for the structure to be erected all loose and decayed parts of the rock must be removed, and the surface made as nearly horizontal as practicable; when the surface is inclined, it may be cut into steps with horizontal

and vertical faces; if holes exist, they may be filled with concrete. In some cases a proper surface for supporting the proposed structure can be secured by covering the rock surface with a layer of concrete, which may vary from a few inches to two or more feet in thickness. (Figs. 39 and 42.)

The *maximum* intensity of the pressure upon a rock foundation should not exceed *one sixth* the crushing strength of the rock for a steady and uniform load, or one tenth the crushing strength for a load due to the weight of the structure plus a varying load such as is caused by wind or earth pressure.

In no case should any portion of the horizontal joints be subjected to tension. The maximum deviation of the centre of pressure from the centre of gravity of the base section, when the section is a symmetrical figure, can be found from the formula

$$x_0 = \frac{I}{Ay}, \text{ (Rankine)};$$

where x_0 = the maximum deviation sought;

I = the moment of inertia of the section relative to an axis perpendicular to the direction in which the maximum deviation is sought;

and y = the distance from the centre of gravity of the section to the edge furthest from the centre of pressure measured along an axis passing through the centre of pressure and the centre of gravity.

Following are the more common sections of foundations with the corresponding values of x_0:

Rectangle.. $A = bh$, $x_0 = \frac{1}{6}b$;

Circle..... $A = \pi r^2$, $x_0 = \frac{1}{8}d$;

Hollow rectangle:

$$A = bh - b'h', \quad x_0 = \frac{b}{6}\left(1 - \frac{b'h'^3}{bh^3}\right) \div \left(1 - \frac{b'h'}{bh}\right);$$

Hol. square. $A = h^2 - h'^2, \quad x_0 = \frac{h}{6}\left(1 + \frac{h'^2}{h^2}\right);$

Hol. circle. $A = \pi(r^2 - r'^2), \; x_0 = \frac{d}{8}\left(1 + \frac{r'^2}{r^2}\right).$

The ultimate compressive strengths of various rocks used in foundations are approximately, for

Granite....................	12800	pounds per square inch.
Sandstone..................	9800	" " " "
Soft sandstone............	3000	" " " "
Strong limestone..........	8500	" " " "
Weak limestone...........	3000	" " " "
Hard red brick............	3000	" " " "
Common brick............	1000	" " " "
Portland cement concrete:		
1 month old............	1000	" " " "
12 months "	6000	" " " "
Rosendale cement concrete:		
6 months old............	1200	" " " "

Foundations upon Earth.

Firm Earth.—Earth which has an angle of repose of at least 27° may be considered as firm, and for foundation purposes requires little preparation other than the excavation of a trench or pit, and making the surface receiving the masonry level. From Table II it is seen that sand, gravel, and damp clay are classed as firm soils; however,

these may become so saturated with water that their angles of repose will become considerably less than 27°, hence precautions must be taken against too much water by draining the ground in the immediate vicinity of the foundation. Particular care must be taken in the case of clay, or sand which will become a kind of quicksand when saturated with water.

Before attempting to design a foundation, the character of the earth must be determined either by test excavations, borings, or from the experience of others. It often happens that from all surface indications the earth appears to be firm, but upon excavating it is found there is a stratum of semi-fluid mud or quicksand underneath; in such cases care must be taken to determine the minimum thickness of the stratum of firm earth, for if too thin it will not be safe to build upon, and then a foundation has to be prepared according to some of the methods described later.

Considering the earth as a homogeneous granular mass, the supporting power at any depth can be computed when the angle of repose ϕ is known. Some practical men object to any theoretical formulas being employed in connection with the determination of the bearing or supporting power of earth, claiming that the assumptions upon which the formulas are based are rarely if ever found in practice. This is probably true to a certain extent, yet the theoretical formulas are upon the safe side, and do not lead to absurd results; in fact, the results obtained by their judicious application agree very well with the practice of the best engineers.

If $p =$ the maximum supporting power per square foot of earth;

$\gamma =$ the weight of one cubic foot of earth;

$\phi =$ the angle of repose;

and x' = the depth of the plane below the surface upon which the maximum supporting power is desired;

then

$$p = x'\gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \text{ (see page 40)}. \quad . \quad . \quad (1)$$

And if p' is the minimum intensity of the pressure upon the earth which is allowable for the stability of the earth and the foundation with its load,

$$p' = x''\gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^2 \text{ (see page 40)}, \quad . \quad . \quad (2)$$

where x'' is the depth of the plane considered below the surface of the earth.

The above equations neglect any friction between the earth and the masonry of the foundation. In deep foundations this is a large factor on the safe side.

If the surface of the earth is level, then $x' = x''$; and further, if the earth is subjected to a uniformly distributed load only the average intensity need be considered.

Equation (2) is considerably different from that given by Rankine, and writers who have followed him, in this, that they consider the minimum intensity allowable to be equal to $x''\gamma$ = the average intensity of the pressure upon a plane at a depth x'' in an unlimited mass. This does not appear to the writer to be a logical treatment of the subject, if the mass has an angle of repose greater than zero, and the maximum intensity allowable be determined as a function of this angle.

According to the assumption of Rankine, it would appear that if a box without a bottom were sunk into a mass of perfectly dry sand it would be filled from the bottom until

the surfaces without and within were at the same level; but this does not take place, and would not even if the sides of the box were frictionless. The sand only fills the box partially, or until the requirements of equation (2) are fulfilled. Hence it seems to the writer that if the maximum intensity is a function of ϕ, the value of the minimum intensity must be also.

From equations (1) and (2) it is evident that the allowable intensity upon the earth of any pressure or load commonly called the supporting power varies *directly as the depth*, as long as ϕ remains unchanged; hence all tables of supporting powers of earth are of little value unless the depth of the foundation upon which they are based is known. Unfortunately this is omitted in most cases, and only the character of the earth is given. The depth to which foundations must be sunk in many localities has a *minimum* value governed by the depth to which frost extends. This is not always true, however, as in Terre Haute, Indiana, frame houses and brick blocks two and one-half stories high are constructed practically upon the surface, the sod only being removed. The width of the foundation is not excessive, but on the contrary narrow. No serious settlement results, owing to the character of the earth, which is very sandy, and will not retain sufficient moisture to permit frost action to heave the structures. The actual load per square foot supported by the soil is about one ton. If x' be taken as one foot, γ as 100 pounds, and p as 2000 pounds, then from equation (1) ϕ is about 39°, which is below the actual value.

The above case, however, may be called an exception to the general rule that all foundations must be sunk below the action of frost, or to a depth of three feet or more according to the locality.

For convenience the values of

$$\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 \quad \text{and} \quad \left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

are given in the following table:

ϕ	$\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2$	$\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$	ϕ	$\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2$	$\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$
0	1.00	1.00	23	5.21	0.19
5	1.42	0.70	24	5.62	0.18
6	1.52	0.66	25	6.07	0.16
7	1.63	0.61	26	6.56	0.15
8	1.75	0.57	27	7.09	0.14
9	1.88	0.53	28	7.67	0.13
10	2.02	0.50	29	8.30	0.12
11	2.16	0.46	30	9.00	0.11
12	2.32	0.43	31	9.76	0.10
13	2.50	0.40	32	10.59	0.09
14	2.68	0.37	33	11.50	0.09
15	2.88	0.35	34	12.51	0.08
16	3.10	0.32	35	13.62	0.07
17	3.33	0.30	36	14.84	0.07
18	3.59	0.28	37	16.18	0.06
19	3.86	0.26	38	17.67	0.06
20	4.22	0.24	39	19.64	0.05
21	4.48	0.22	40	21.16	0.05
22	4.83	0.21			

Having determined upon the depth to which it is expedient to extend the foundation, a *minimum* value of ϕ must be assumed from a knowledge of the earth, and then the allowable bearing or supporting power can be found from equations (1) and (2); or if the supporting power is assumed, the minimum depth to which the foundation must be sunk can be found from the same equations.

The proper proportions of the foundation are most easily obtained from the following equations, which are deduced

for a few of the ordinary forms and conditions. All masonry foundations are usually trapezoidal in section, and hence formulas based upon this form can be applied to stepped foundations without serious error.

CASE I. *Given a uniformly distributed load to be supported by symmetrical trapezoidal foundation sunk to a known depth, to determine the minimum width of the base of the foundation.*

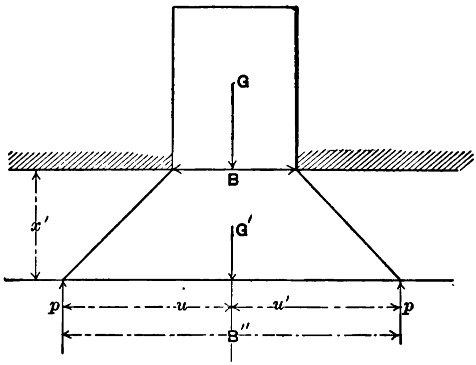


FIG. 35.
Section of Wall and Foundation.

Let G = the total weight to be supported less that of the foundation;

G' = G + weight of the foundation;

and B'' = minimum breadth of the foundation.

Assuming x', the value of p is

$$p = x'\gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2.$$

From the figure

$$G' = G + W\frac{B + B''}{2}x' = B''p;$$

or

$$B'' = \frac{2G + BWx'}{2p - Wx'}.$$

The above formula applies to a wall one foot long.—In case of an isolated pier, the value of x' can be found as above. B'' may be assumed and a rough calculation made to determine if the average pressure upon the earth is equal to or less than p. A second trial usually determines the proper value for B''. The exact formula for the determination of the dimensions of a square or rectangular foundation with stepped sides is an equation of the second degree.

Ex. 1. A trapezoidal foundation 5 feet broad on top has to support 50,000 pounds per lineal foot in length, in earth having a minimum angle of repose of 30°. The maximum depth to which the foundation is to be sunk is 5 feet; determine B'' and p, when $\gamma = 100$ pounds and $W = 150$ pounds.

From (1)

$$p = 5 \cdot 100 \cdot 9 = 4500 \text{ pounds—say } 4000;$$

then

$$B'' = \frac{100000 + 3750}{8000 - 750} = 14.3;$$

or the proper width of the base is about 14.5 feet.

Ex. 2. A cast-iron plate, 2 feet square under a column, transmits a load of 20,000 pounds to a masonry foundation 3 feet square. How deep must this be sunk in earth when $\phi = 30°$, $\gamma = 100$ pounds, and $W = 150$ pounds?

Neglecting the weight of the masonry in the foundation, the intensity of the pressure upon the earth is about 2200 pounds; then from (1) x' = about 2.5 feet—say 3 feet.

The actual intensity of the pressure upon the earth is now $\frac{20,000 + 4050}{9} = 2670$ pounds. Substituting this value of p in (1) and solving for x', its value is 2.85 feet; hence 3 feet is the required depth of the foundation.

The weight of the earth supported by the masonry of the foundation is neglected.

CASE II. *Unsymmetrical distribution of pressure upon the base of a foundation.*

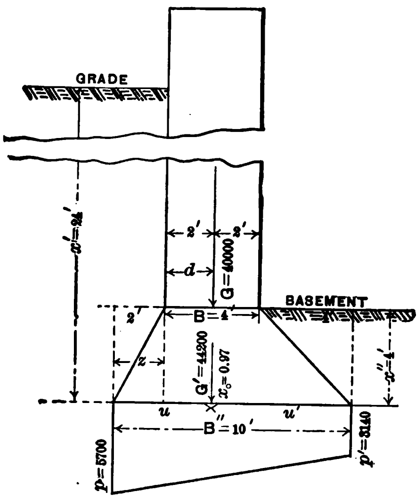


FIG. 36.
Section of Wall and Foundation.

One of the many examples of pressure unevenly distributed upon the bed of a foundation is the case of an outside wall of a building located very near the property line and circumstances prevent encroaching upon the neigh-

boring property to any great extent. Here two conditions must be fulfilled. The maximum intensity of the pressure p, Fig. 36, must not be greater than the supporting power of the earth at the depth x', and the minimum intensity p' must not be so small that the earth having a depth x'' may tend to heave the foundation.

Let p_0 = the *average* intensity of the pressure upon the base. Then

$$p_0 = \frac{G'}{B''} = \frac{p + p'}{2}, \quad p = x'\gamma \left\{ \frac{1 + \sin\phi}{1 - \sin\phi} \right\}^2.$$

But

$$G' = G + \frac{B + B''}{2} x'' W.$$

Therefore

$$B'' = \frac{2G + BWx''}{2p_0 - Wx''};$$

in which x'' is determined from the equation

$$p' = x''\gamma \left\{ \frac{1 - \sin\phi}{1 + \sin\phi} \right\}^2.$$

It is thus possible to determine B'' quite easily, but the value of the offset z so that p and p' shall have their proper values must be either found by trial or computation. Since one or two trials are sufficient to determine z, the formula will not be given here.

Ex. 3. In Fig. 36, page 75, let $G = 40{,}000$ pounds, $B = 4$ feet, $d = 2$ feet, $x' = 24$ feet, and $x'' = 4$ feet. If the thrust of the earth be neglected, what must be the width of the base of the foundation, so that the average pressure per unit area shall not exceed 4800 pounds, and the maximum 7000 pounds, when $\gamma = 100$, $W = 150$,

$\phi = 30°$? The bulk of the foundation to be on the right of the centre of the wall.

First determine the allowable intensities,

$$
\begin{aligned}
\max p &= x'\gamma(9) &&= 2400 \times 9 &&= 21600 \text{ pounds.}\\
\min &= x'\gamma(0.11) &&= 2400 \times 0.11 &&= 264 \quad \text{"}\\
\max p' &= x''\gamma(9) &&= 400 \times 9 &&= 3600 \quad \text{"}\\
\min &= x''\gamma(0.11) &&= 400 \times 0.11 &&= 44 \quad \text{"}
\end{aligned}
$$

From the formula on page 76

$$B'' = \frac{2G + BWx''}{2p_0 - Wx''} = \frac{82400}{9000} = 9.15 \text{ feet.}$$

Take 10 feet as the value of B''; then the weight of the masonry in the foundation is 4200 pounds, and

$$p_0 = \frac{44200}{10} = 4420.$$

By graphics or by moments, assuming $z = 2$ feet, the resultant pressure cuts the base 0.97 foot from the centre, and hence $p = 5700$ pounds and $p' = 3140$ pounds.

The above width of base and the intensities just obtained satisfy all the conditions of the problem, though the value of z could be decreased a little, increasing the intensity at the toe and decreasing that at the heel.

Projection of Footing-courses.—Where masonry foundations are stepped as is the usual custom, the proper offset for each course may be determined as follows, by considering each offset as a cantilevered beam of stone *uniformly loaded:*

Let o = the offset of any particular course;

p_0 = the intensity of the pressure upon the base of the course;

t = the thickness of the course;
R = the modulus of rupture of the material; and
F = the factor of safety.

Then

$$p_{\circ}\frac{o^2}{2} = \frac{1}{6}\frac{R}{F}t^2,$$

or

$$o = t\sqrt{\frac{R}{F}\,\frac{1}{3p_{\circ}}}.$$

In case the intensity of the pressure is not uniform, but varies uniformly from one side to the other, the quantity $p_{\circ}$ may be replaced by p, the maximum intensity for the offset on the side having the greater pressure, and by p', the minimum intensity for the steps or offsets on the side of the lesser pressure: in the first case the factor of safety will be slightly increased and in the second decreased.

The above formula is applicable only when the stones project less than half their length and when thoroughly well laid in cement mortar.

The table on the following page is given by Prof. Baker.

Other factors remaining the same, the offsets vary *directly* as the square roots of the moduli of rupture and inversely as the factors of safety, so that the above table can be applied for any values of R and F by simple proportion.

Foundations upon Soft Earth.—When a foundation must be placed upon soft earth which offers no particular difficulties other than the requirement of broadness or depth of the excavation, considerable expense can be avoided by excavating the soft material and replacing it by firm material, or by driving short piles spaced about

three feet on centres, commencing at the outer limits of the foundation and working towards the centre, and thus compressing the earth; sometimes holes are bored and filled with sand, making sand-piles, etc. The proper depth and spread of such foundations can be found from formulas (1) and (2) by including the prepared earth as a portion of the foundation.

SAFE OFFSETS FOR MASONRY FOOTING-COURSES,

IN TERMS OF THE THICKNESS OF THE COURSE, USING 10 AS A FACTOR OF SAFETY.

Kind of Stone.	R in Lbs. per Sq. In.	Offsets for a Pressure, in Tons per Sq. Ft., on the Bottom of the Course of Masonry.		
		0.5	1.0	1.5
Bluestone flagging	2700	3.6	2 6	1.8
Granite	1800	2.9	2.1	1.5
Limestone	1500	2.7	1.9	1.3
Sandstone	1200	2.6	1.8	1.3
Slate	5400	5.0	3.6	2.5
Best hard brick	1500	2.7	1.9	1.3
Hard brick	800	1.9	1.4	0.8
Concrete { 1 Portland, 2 sand, 3 pebbles } 10 days old	150	0.8	0.6	0.4
Concrete { 1 Rosendale, 2 sand, 3 pebbles } 10 days old	80	0.6	0.4	0.3

In case the earth has sufficient water to keep the foundation damp, a very excellent foundation upon soft earth is a platform of timber composed of heavy sticks laid close together in layers, every alternate layer being right-angled with that adjacent, and thoroughly driftbolted together. Another method is to form a grillage of the timbers and fill the spaces around the sticks with concrete.

In dry soft earth the timber platform may be replaced by a bed of concrete, which is more durable, but not as elastic. Recently the combination of iron or steel beams with concrete has been successfully employed for foundations upon soft earth in Chicago.

The safe projection of the timber platform or one of concrete beyond the masonry can be found by the formula already given.

The safe projection of iron or steel beams can be found as follows:

Let I = the moment of inertia of the section;
h = the depth of the beam;
p_e = the intensity of the pressure upon the bed of the foundation transmitted to the beam;
R = the modulus of rupture of the material composing the beam;
and F = the factor of safety.

Then

$$p_e \frac{o^2}{2} = 2 \frac{RI}{Fh}$$

or

$$o = 2 \sqrt{\frac{R}{F} \frac{1}{p_e h} I}$$

In case the pressure upon the base of the foundation is not uniform, the method outlined for masonry offsets can be applied in proportioning the offsets of steel or iron beams.

The following table of the safe projections of steel I beams is given in Carnegie's Pocket Companion.

TABLE

GIVING SAFE LENGTHS OF PROJECTIONS "*o*" IN FEET (SEE ILLUSTRATION), FOR "*s*" = 1 FOOT AND VALUES OF "p_0" RANGING FROM 1 TO 5 TONS.

Depth of Beam, in.	Weight per Foot, lbs.	*b* (Tons per Square Foot.)										
		1	1¼	1½	2	2¼	2½	3	3½	4	4½	5
20	80	14.0	12.5	11.5	10.0	9.0	9.0	8.0	7.5	7.0	6.5	6.0
20	64	12.5	11.0	10.0	8.5	8.0	8.0	7.0	6.5	6.0	6.0	5.5
15	80	12.0	10.5	9.5	8.5	8.0	7.5	7.0	6.5	6.0	5.5	5.0
15	60	10.5	9.5	8.5	7.5	7.0	6.5	6.0	5.5	5.5	5.0	5.0
15	50	9.5	8.5	8.0	7.0	6.5	6.0	5.5	5.0	5.0	4.5	4.5
15	41	8.5	8.0	7.0	6.0	6.0	5.5	5.0	4.5	4.5	4.0	4.0
12	40	8.0	7.0	6.5	5.5	5.5	5.0	4.5	4.0	4.0	3.5	3.5
12	32	7.0	6.5	5.5	5.0	4.5	4.5	4.0	4.0	3.5	3.5	3.0
10	33	6.5	6.0	5.5	4.5	4.5	4.0	4.0	3.5	3.5	3.0	3.0
10	25.5	5.5	5.0	4.5	4.0	4.0	3.5	3.5	3.0	3.0	2.5	2.5
9	27	5.5	5.0	4.5	4.0	4.0	3.5	3.5	3.0	3.0	2.5	2.5
9	21	5.0	4.5	4.0	3.5	3.5	3.0	3.0	2.5	2.5	2.5	2.0
8	22	5.0	4.5	4.0	3.5	3.5	3.0	3.0	2.5	2.5	2.5	2.0
8	18	4.5	4.0	3.5	3.0	3.0	3.0	2.5	2.5	2.0	2.0	2.0
7	20	4.5	4.0	3.5	3.0	3.0	3.0	2.5	2.5	2.0	2.0	2.0
7	15.5	4.0	3.5	3.0	2.5	2.5	2.5	2.0	2.0	2.0	2.0	1.5
6	16	3.5	3.0	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.5	1.5
6	13	3.0	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.5	1.5	1.5
5	18	3.0	2.5	2.5	2.0	2.0	2.0	1.5	1.5	1.5	1.5	1.5
5	10	2.5	2.5	2.0	2.0	1.5	1.5	1.5	1.5	1.5	..	..
4	10	2.5	2.0	2.0	1.5	1.5	1.5	1.5	..	..	..	..
4	7.5	2.0	2.0	1.5	1.5	1.5	1.5	..	..	..	..	..

Above table applies to *steel* beams. Values given based on extreme fibre stresses of 16,000 pounds per square inch.

Pile Foundation.—Pile foundations are employed in all kinds of earth, sometimes to save expense and sometimes because nothing else appears to be as good. In localities where the earth is uncertain in its character the use of

piles enables the engineer to put in a foundation which he feels sure is safe, as a single pile thirty feet long will support several tons even when driven into mud, the load in this case being carried almost entirely by the friction of the

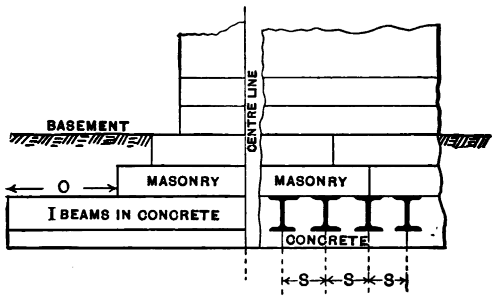


FIG. 37.

mud upon the surface of the pile. If the pile is driven through the mud to a solid stratum below, then the pile acts as a column more or less supported its entire length, and consequently able to carry a very great load.

Piles are usually spaced about three feet on centres, and the tops firmly bedded in a layer of concrete or stayed by a grillage of timber or by a combination of these methods, the object being to thoroughly and evenly distribute the load to be supported.

The supporting power of a pile in a given earth can be found in the following manner:

Let G' = the total load to be supported by the pile, including the weight of the pile;

p_0 = the intensity of the pressure upon the bottom of the pile;

A = the superficial area of the pile in contact with the earth;

and $\quad f =$ a factor depending upon the friction resistance of a unit area of the surface of the pile.

Then for a pile having a diameter of d

$$* \; G' = \frac{\pi d^2}{4} p_0 + fA.$$

But

$$p_0 = x'\gamma \left\{ \frac{1 + \sin\phi}{1 - \sin\phi} \right\}^2 \quad \text{and} \quad A = \pi d x'.$$

$$\therefore \quad x' = \frac{G'}{\gamma \frac{\pi d^2}{4} \left\{ \frac{1 + \sin\phi}{1 - \sin\phi} \right\}^2 + f\pi d.}$$

For practical purposes this may be written

$$x' = \frac{G'}{\gamma \left\{ \frac{1 + \sin\phi}{1 - \sin\phi} \right\}^2 + 3f.}$$

For convenience this may be further simplified for special cases.

The following values of f have recently been given by W. M. Patton, based upon his own and the experience of others:

In very soft silt or liquid mud,	$f = 150$	pounds	per sq. ft.
In ordinary clay or earth (dry),	$f = 300$	"	" " "
" " " " " (wet),	$f = 150$	"	" " "
In compact hard clay,	$f = 300$	"	" " "
In sand, or sand and gravel,	$f = 500$	"	" " "

* This formula was suggested by reading W. M. Patton's article on piles in his "Practical Foundations."

For the silt of swamps, muds, etc., ϕ is very nearly if not quite zero. So as to be on the side of safety, ϕ will be taken as zero, $f = 150$ pounds. Then

$$x' = \frac{G'}{120 + 450} = \frac{G'}{570}, \text{ say } \frac{G'}{600}.$$

a very simple formula.

For moist clay, $\phi =$ about $17°$, $\gamma = 120$ pounds, and $f = 150$ pounds. Then

$$x' = \frac{G'}{120 \,.\, 3\tfrac{1}{3} + 450} = \frac{G'}{850}.$$

For dry, compact sand, $\phi = 27°$, $\gamma = 106$ pounds, and $f = 500$ pounds. Then

$$x' = \frac{G'}{107.7 + 1500} = \frac{G'}{2249}, \text{ say } \frac{G'}{2300}.$$

In a similar manner the safe load for a pile in any earth can be determined when ϕ and f are known. These quantities must be the result of experiment. Any formula which does not include these factors is incomplete, and neglects the factors upon which the supporting power of the pile directly depends.

The character of the earth through which the pile is to be driven can be determined by borings, and thus ϕ and γ determined upon.

The value of f can be found by studying the behavior of piles already driven in similar earth. Thus it appears that the above formula must be as accurate in results and as safe in application as the majority of the formulas used by engineers in proportioning structures.

The formula is independent of the means by which the pile is driven, as ought to be the case, since very often piles are sunk by water-jets, or even by working them backwards and forward, making the formulas depending upon the weight of a driving-hammer, its fall, and the penetration of the pile during the last few blows useless. Two of the most simple of the many formulas of this class are those of Trautwine and the *Engineering News*, viz.:

$$G' = \frac{3\sqrt{h} \times W \times 0.0268}{2(1 + a)} \text{ (Trautwine's);}$$

$$G' = \frac{2Wh}{a + 1} \text{ (\textit{Eng. News});}$$

where G' = the safe load;
W = the weight of the hammer in pounds;
h = the fall of the hammer in inches;
and a = the average penetration of the pile in inches during the last few blows.

Screw-pile.—Screw-piles are usually round, and have at the bottom a cast or wrought iron screw. The piles are of wood, cast iron, or wrought iron. The diameter of the screw is from two to eight times the diameter of the pile, and its pitch from one fourth to one half its diameter. The screw seldom has but one turn. The piles are sunk by turning them by means of levers or by power. (Fig. 45.)

The load which the pile will carry depends principally upon the supporting power of the earth at the depth of the screw and the area of the screw, though in all cases there is more or less frictional resistance upon the surface of the pile proper. If x' is the depth of the screw and p_0 the

allowable intensity of the pressure upon the earth at that depth, then

$$p_0 = \frac{x'}{\gamma}\left\{\frac{1 - \sin\phi}{1 + \sin\phi}\right\}^2.$$

Screw-piles can be advantageously employed for supporting structures above water where the upper ends of the piles can be used as columns. They are chiefly employed in light-house construction.

Sheet-piles.—Sheet-piles are usually of wood in the form of planks, and are driven as closely together, edge to edge, as possible, the object being to form a water-tight barrier.

To make the joints tight the planks are oftened tongued and grooved. A patent sheet-pile is formed by bolting together three planks of equal width, so that the middle plank will form the tongue on one side and the outside planks the groove on the other side. Sheet-piles are also employed to confine soft earths.

FOUNDATIONS UNDER WATER AND DEEP FOUNDATIONS.

Foundations under water differ in general but little from those upon dry earth, the effect of water, ice, etc., upon the structure, however, constitute additional problems to be solved for each locality.

A few of the various methods employed in placing foundations under water or at great depths will be very briefly described.

Coffer-dams.—A coffer-dam is merely a tight wall surrounding the locality where the foundation is to be placed, excluding water from the enclosure, which can be pumped dry and the surface prepared to receive the foundation.

In quiet and shallow water the dam may be made of earth, or sheet-piles banked with earth.

In deep water large piles are driven every few feet in two rows around the site, to which horizontal timbers are bolted, acting as guides and supports to a double row of sheet-piles, between which is placed puddled earth. To prevent bending, the large piles are cross-tied with bolts.

The space enclosed should be somewhat larger than required by the foundation, to allow room for materials, etc. (Fig. 46.)

Timber Cribs.—A timber crib is a box built of large timbers and divided into cells by cross partitions. The joints and splices of the timbers employed are arranged so that walls and partitions are thoroughly tied together. In

case a tight wall-crib is wanted the timbers may be dapped one fourth their depth on both sides or halved together. Cribs are built in the shape best suited to the purpose for which they are to be used. They are usually constructed at some convenient point near the site of the foundation, and then towed to the place where they are to be sunk. In constructing the crib a few of the cells are planked near the bottom. These are filled with stone until the crib sinks to the surface previously prepared to receive it. The other cells are now filled with stone and the regular masonry commenced. Sometimes the top of the crib is planked over before the masonry is started. (Fig. 44.)

The surface which is to receive the crib may be soft mud, riprap, rock, or piles. The crib is allowed to sink into the mud and to rest upon riprap which has been levelled. If the surface is level rock, the crib is merely sunk; but if the rock is uneven, it is either levelled or the crib is sunk until it just touches rock at some point, when riprap is thrown around and under the crib.

Timber cribs are extensively employed in various classes of engineering works for both temporary and permanent structures.

In permanent structures the timbers supporting masonry, etc., should always be under water.

Timber cribs are sometimes used as coffer-dams by making the outside cells water-tight. The crib is sunk into the mud, or the bottom edges banked with earth, etc., until the interior can be kept dry by pumping.

Open Caissons.—An open caisson is a strong water-tight box which is floated to the site of the foundation and sunk to its place by the masonry proper, which is built inside the box. After the bottom has reached its position and the top of the masonry is above water, the sides are removed,

leaving the bottom of the box as a platform supporting the masonry. The surface to receive an open caisson is prepared by dredging, throwing in riprap, driving piles, etc., as best suits the locality. (Fig. 47.)

Cushing Cylinder Piers.—A cluster of piles is first driven as closely together as possible, and their tops thoroughly bolted one to the other. Then an iron cylinder is placed around the cluster and built up in sections until the top is above water. Then the cylinder is made to sink by dredging out the material inside by water-jets, by disturbing the material around the edges, etc., until a desired depth is reached, sections being bolted to the top of the cylinder as needed. The cylinder is now filled with concrete to the top and covered with an iron cap which receives the load to be carried. The size and number of cylinders employed depends upon the superstructure.

For ordinary bridges two cylinders cross-braced form a pier.

The supporting power depends upon the piles principally, though the friction upon the outside of the cylinders offers some resistance to settlement.

Pneumatic Caissons.—A pneumatic caisson is essentially an air-tight box with the open side imbedded in earth, from which the air is pumped to allow the box to sink or into which air is pumped to prevent sinking. In water the caisson usually carries a water-tight timber crib, which in turn supports a timber coffer-dam, the crib enabling the structure to be loaded with stone according to the requirements of the sinking operation, and the coffer-dam keeping the water out near the surface. Various combinations of caisson, crib, and coffer-dam are made, however, to suit conditions. (Fig. 48.)

The ordinary method of sinking caissons is to pump in

enough air to exclude water from the chamber, while laborers dig out the material over the surface and near the edges of the chamber, this material being removed by various methods such as pumps, lifts, etc. When sufficient material has been removed, all the laborers leave the caisson, leaving one man only who watches for leaks; the air-pressure is then lowered a little, and the caisson with its superstructure sinks. This process is repeated until a solid foundation is reached, when the caisson is filled with concrete, as also are the cribs, etc., if any, above the caisson.

TYPES OF EXISTING FOUNDATIONS.

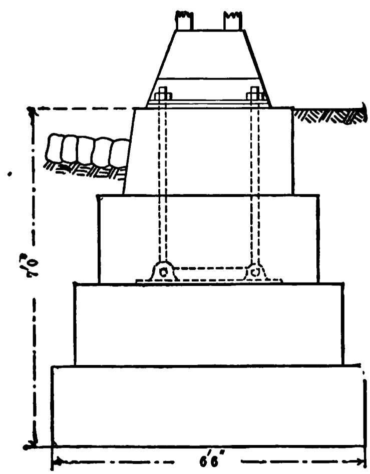


FIG. 38.

Concrete Pier used as Foundation for Elevated Railroad Columns (*Engineering and Building Record*, Sept. 14, 1895.)

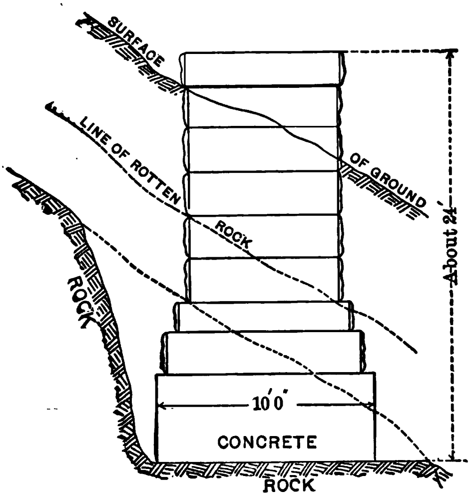


FIG. 39.

Elevation of Masonry Pier with Bottom Course of Concrete. Illustrating the removal of rotten rock and the levelling of the rock surface. (Marent Gulch Viaduct, N. P. R. R.; *Trans. Am. Soc. C. E.*, Sept., 1891.)

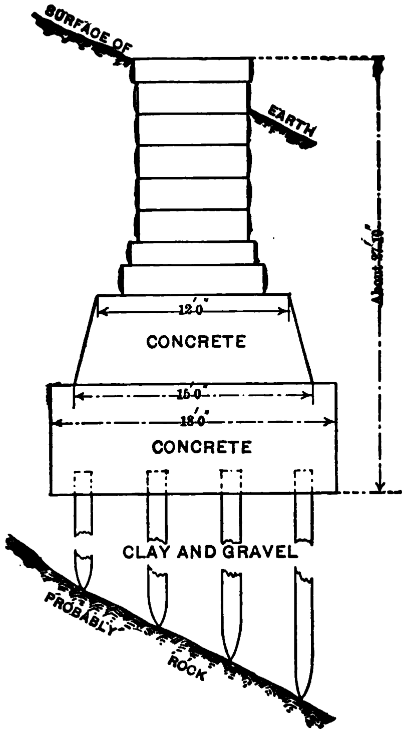


FIG. 40.

Elevation of another Pier of the Marent Viaduct Foundations. Showing the application of piles and concrete to obtain a solid foundation.

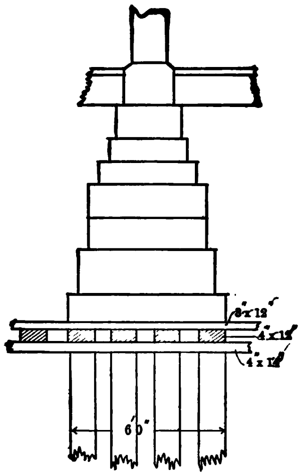


FIG. 41.

Elevation of a Pier in the Foundation of a Chicago Grain Elevator. Illustrating the use of piles and a wooden platform in soft ground. Piles are from 20 to 40 feet long, and reach hardpan. Twelve piles are placed under each post, and each pile supports a load of about 22 tons. (*Engineering and Building Record*, Nov. 12, 1895.)

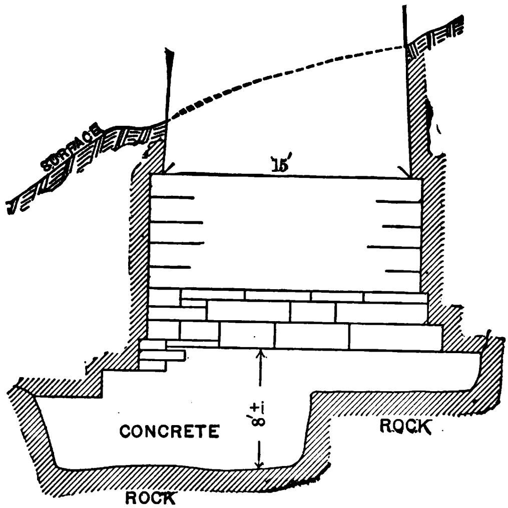


FIG. 42.

End Elevation of Masonry Pier supporting Stone Arches of Washington Bridge. Illustrating the use of concrete to level the rock surface to receive masonry.

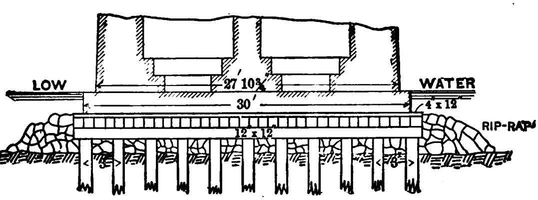


FIG. 43.

Section through Centre of Foundation of Pivot Pier of Grand Forks Bridge. Illustrating the use of piles, wooden platform, and riprap. (*Baker.*)

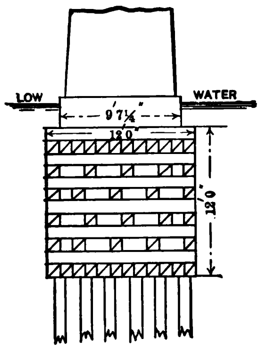


FIG. 44.

End Elevation of Foundation of Pier of Croix River Bridge. Illustrating the use of timber crib and piles. (*Baker.*)

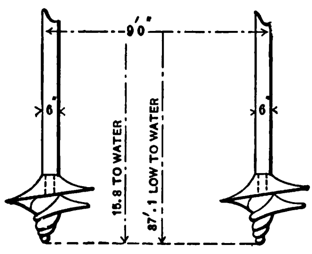


FIG. 45.

Mobile River Bridge Piers. Composed of two rows of screw-piles, about 9 feet centre to centre, with piles spaced about 8 feet apart. (See *Engineering News*, vol. xiii. p. 210.)

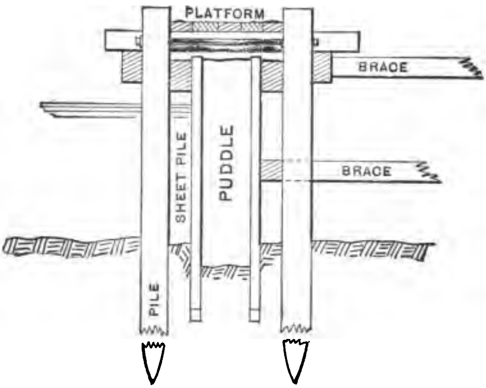


FIG. 46.
Sketch showing Cross-section of Coffer-dam.

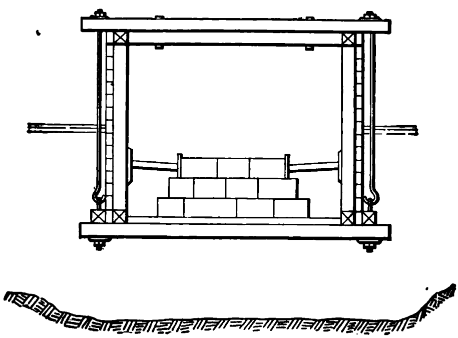
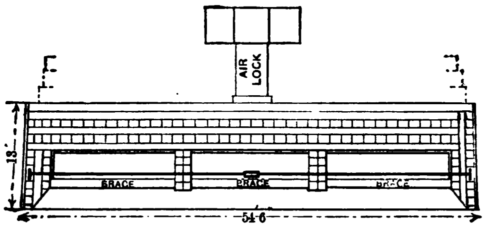


FIG. 47.
Sketch showing Essential Features of Open Caisson.



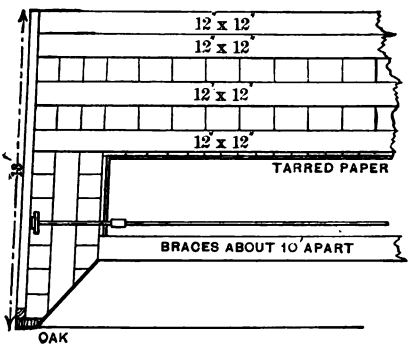


FIG. 48.

Section of One of the Caissons employed in the Foundations of the Piers for the Washington Bridge.

REFERENCES.

EARTH-PRESSURE AND RETAINING-WALLS.

A brief outline of the theories advanced by the following writers can be found in "*Neue Theorie des Erddruckes*," *Dr. E. Winkler, Wien*, 1872:

D'Antony,	Hoffmann,	Poncelet,
Andé,	Holzhey,	Prony,
Andoy,	de Lafont,	Rankine,
Belidor,	Levi,	Rebhann,
Blaveau,	de Köszegh Martony,	Rondelet,
Bullet,	Maschek,	Saint-Guilhem,
Considère,	Mayniel,	Saint-Venant,
Coulomb,	Mohr,	Sallonnier,
Couplet,	Montlong,	Scheffler,
Culmann,	Moseley,	Trincaux,
Français,	Navier,	Vauban,
Gadroy,	Ortmann,	Winkler,
Gauthey,	v. Ott,	Woltmann.
Hagen,	Persy,	

AUDÉ. Poussée des Terres. Nouvelles expériences sur la poussée des terres. Paris, 1849.

BAKER-CURIE. Note sur la brochure de M. B. Baker théorie. Annales des Ponts et Chaussées, pp. 558–592, 1882.

—— The actual lateral pressure of earthwork. Van Nostrand's Magazine, xxv, 1881; also Van Nostrand's Science Series, No. 56.

BOUSSINESQ. Complément à de précédentes notes sur la poussée des terres. *Annales P. et C., 1884.

BOUSIN. Equilibrium of pulverulent bodies. The equilibrium of earth when confined by a wall. †Van N., 1881.

CAIN. Modification of Weyrauch's Theory. Van N., 1880.

—— Earth-pressure. Modification of Weyrauch's Theory. Criticism of Baker's articles. Van N., 1882.

—— Uniform cross-section, and T abutments: their proper proportions and sizes, deduced from Rankine's general formulas. Van N., 1872.

—— Practical designing of retaining-walls. Van N. Science Series, No. 3, 1888.

CHAPERON. Observations sur le mémoire de M. de Sazilly (1851). Stabilité et consolidation des talus. Annales P. et C., 1853.

CONSIDÈRE. Note sur la poussée des terres. Annales P. et C., 1870.

COUSINERY. Détermination graphique de l'épaisseur des murs de soutènement. Annales P. et C., 1841.

DE LAFONT. Sur la poussée des terres et sur les dimensions à donner, suivant leurs profils, aux murs de soutènement et de reservoirs d'eau. Annales P. et C., 1866.

DE SAZILLY. Sur les conditions d'équilibre des massifs de terre, et sur les revêtements des talus. Annales P. et C., 1851.

EDDY. Retaining-walls treated graphically. Van N., 1877.

FLAMANT. Note sur la poussée des terres. Annales P. et C., 1882.

—— Résumé d'articles publiés par la Société des Ingénieures Civils de Londres sur la poussée des terres. Annales P. et C., 1883.

---

* Annales des Ponts et Chaussées.

† Van Nostrand's Magazine.

FLAMANT. Note sur la poussée des terres. Annales P. et C., 1872.

—— Mémoire sur la stabilité de la terre sans cohésion par W. J. Macquorm Rankine (Extrait 1856–57). Annales P. et C., 1874.

GOBIN. Détermination précis de la stabilité des murs de soutènement et de la poussée des terres. Annales P. et C., 1883.

GOULD. Theory of J. Dubosque. Van N., 1883.

—— Designing. Van N., 1877.

JACOB. Practical designing of retaining-walls. Van N., 1873; also Van N. Science Series, No. 3.

JACQUIER. Note sur la détermination graphique de la poussée des terres. Annales P. et C., 1882.

KLEITZ. Détermination de la poussée des terres et établissement des murs de soutènement. Annales P. et C., 1844.

LAGREUE. Note sur la poussée des terres avec ou sans surcharges. Annales P. et C.. 1881.

L'ÈVEILLÉ. De l'emploi des contre-forts. Annales P. et C. 1844.

LEYGUE. Sur les grands murs de soutènement de la ligne de Mezamet a Bédarieux. Annales P. et C., 1887.

—— Nouvelle recherche sur la poussée des terres et le profil de revêtement le plus économique. Annales P. et C., 1885.

MERRIMAN. On the theories of the lateral pressure of sand against retaining walls. (School of Mines Quarterly.) Engineering News, 1888.

—— The theory and calculation of earthwork. Engineering News, 1885.

REBHANN. Theorie des Erddruckes und der Futtermauern. Wien, 1870 and 1871.

SAINT-GUILHEM. Sur la poussée des terres avec ou sans surcharge. Annales P. et C., 1858.

SCHEFFLER-FOURNIE. Traité de la stabilité des constructions. Paris, 1864.

TATE. Surcharged and different forms of retaining walls. Van N., 1873; also Van N. Science Series, No. . Also published by E. & F. N. Spon.

THORNTON. Theory. Van N., 1879.

FOUNDATIONS.

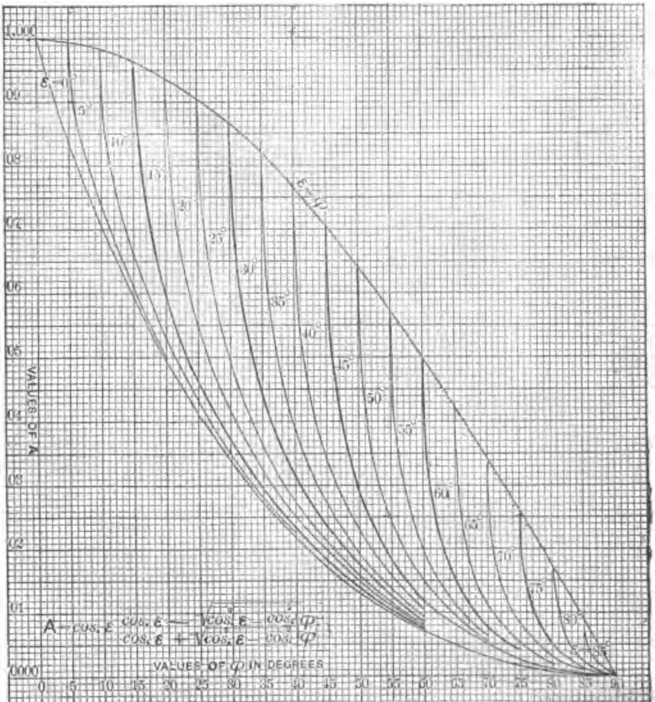
BAKER. A treatise on masonry construction. John Wiley & Sons, N. Y.

PATTON. A practical treatise on foundations. John Wiley & Sons, N. Y.

—— A treatise on civil engineering. John Wiley & Sons, N. Y.

For articles in engineering periodicals the reader is referred to a "Descriptive Index of Current Engineering Literature" (1884–1891), published by the Board of Managers of the Association of Engineering Societies.

DIAGRAM I.



1,000
.09
.08
.07
.06
.05
.04
.03
.02
.01
.0000
VALUES OF A
VALUES OF φ IN DEGREES
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
ε = 0°
5°
10°
15°
20°
25°
30°
35°
40°
45°
50°
55°
60°
65°
70°
75°
80°
85°
ε = φ
$A = \cos.\varepsilon \, \frac{\cos.\varepsilon - \sqrt{\cos.^2\varepsilon - \cos.^2\varphi}}{\cos.\varepsilon + \sqrt{\cos.^2\varepsilon - \cos.^2\varphi}}$

TABLES.

Table I contains the crushing-strengths and the average weights of stone likely to be used in the construction of retaining-walls and foundations; also the average weights of different earths.

Table II contains the coefficients of friction, limiting angles of friction, and the reciprocals of the coefficients of friction for various substances.

Tables III, IV, and V contain the values of the coefficients [see equation (1′)] (B), (C), (D) and (E), where

$$(B) = \frac{\cos(\epsilon - \alpha)}{\cos^2\alpha \cos\epsilon}, \quad (C) = \sin^2\alpha, \quad (D) = \left\{ \frac{\cos(\epsilon - \alpha)}{\cos\epsilon} \right\}^2$$

and

$$(E) = 2 \sin\alpha \sin\epsilon \frac{\cos(\epsilon - \alpha)}{\cos\epsilon}.$$

The tables were computed with a Thacher calculating instrument and checked by means of diagrams. It is believed that they are correct to the second place of decimals; an error in the third place of decimals does not affect the results for practical purposes.

Table VI contains the natural sines, cosines and tangents.

TABLE I.

VALUES OF W.

Name of Substance.	Crushing Lds. in tons per sq. ft.	Average weight in lbs. per cu. ft.
Alabaster	....	144
Brick, best pressed	40 to 300	150
" common hard	....	125
" soft inferior	....	100
Chalk	20 to 30	156
Cement, loose	....	49.6 to 102
Flint	....	162
Feldspar	....	166
Granite	300 to 1200	170
Gneiss	....	168
Greenstone, trap	....	187
Hornblende, black	....	203
Limestones and Marbles, ordinary	250 to 1000	{ 164.4 / 168
Mortar, hardened	....	103
Quartz, common	....	165
Sandstone	150 to 550	151
Shales	....	162
Slate	400 to 800	175
Soapstone	....	170

VALUES OF γ.

Name of Substance.	Average weight in lbs. per cu. ft.
Earth, common loam, loose	72 to 80
" " " shaken	82 " 92
" " " rammed moderately	90 " 100
Gravel	90 " 106
Sand	90 " 106
Soft flowing mud	104 " 120
Sand perfectly wet	118 " 129

TABLE II.

* ANGLES AND COEFFICIENTS OF FRICTION.

	$\tan \phi$.	ϕ	$\frac{1}{\tan \phi}$
Dry masonry and brickwork	0.6 to 0.7	31° to 35°	1.67 to 1.43
Masonry and brickwork with damp mortar.......	0.74	36½°	1.35
Timber on stone...........	about 0.4	22°	2.5
Iron on stone	0.7 to 0.3	35° to 16⅔°	1.43 to 3.33
Timber on timber..........	0.5 " 0.2	26½° " 11⅓°	2 " 5
Timber on metals..........	0.6 " 0.2	31° " 11⅓°	1.67 " 5
Metals on metals...........	0.25 " 0.15	14° " 8½°	4 " 6.67
Masonry on dry clay.......	0.51	27°	1.96
" " moist clay.....	0.33	18¼°	3.
Earth on earth	0.25 to 1.0	14° to 45°	4 to 1
Earth on earth, dry sand, clay, and mixed earth....	0.38 " 0.75	21° " 37°	2.63 " 1.33
Earth on earth, damp clay .	1.0	45°	1
Earth on earth, wet clay. .	0.31	17°	3.23
Earth on earth, shingle and gravel..................	0.81	39° to 48°	1.23 to 0.9

* From Rankine's Applied Mechanics.

TABLE III.

ϵ	$a = 5°$	$a = 6°$	$a = 7°$	$a = 8°$	$a = 9°$
	(B)	(B)	(B)	(B)	(B)
0	1.004	1.005	1.007	1.010	1.012
5	1.012	1.015	1.018	1.022	1.026
10	1.019	1.024	1.029	1.035	1.040
15	1.027	1.034	1.041	1.048	1.055
20	1.036	1.044	1.052	1.062	1.071
25	1.045	1.055	1.065	1.076	1.088
30	1.055	1.066	1.079	1.092	1.105
35	1.065	1.079	1.094	1.109	1.124
40	1.078	1.094	1.111	1.129	1.147
45	1.093	1 111	1.131	1.152	1.173
	(C)	(C)	(C)	(C)	(C)
	0.008	0.011	0.015	0.019	0.024

TABLE IV.

ϵ	$a = 5°$	$a = 6°$	$a = 7°$	$a = 8°$	$a = 9°$
	(D)	(D)	(D)	(D)	(D)
0	0.992	0.989	0.985	0.981	0.976
5	1.008	1.008	1.006	1.005	1.003
10	1.023	1.026	1.028	1.030	1.031
15	1.040	1.046	1.051	1.056	1.060
20	1.057	1.066	1.075	1.084	1.092
25	1.075	1.089	1.102	1.114	1.125
30	1.096	1.113	1.130	1.147	1.163
35	1.118	1.140	1.164	1.183	1.204
40	1.144	1.172	1.199	1.226	1.253
45	1.174	1.208	1.242	1.276	1.309

TABLE V.

ϵ	$a = 5°$	$a = 6°$	$a = 7°$	$a = 8°$	$a = 9°$
	(E)	(E)	(E)	(E)	(E)
0	0	0	0	0	0
5	0.015	0.018	0.021	0.024	0.027
10	0.031	0.037	0.043	0.049	0.055
15	0.046	0.055	0.065	0.074	0.083
20	0.061	0.074	0.086	0.099	0.112
25	0.076	0.092	0.108	0.124	0.140
30	0.091	0.110	0.130	0.149	0.169
35	0.106	0.128	0.151	0.174	0.197
40	0.120	0.145	0.172	0.198	0.225
45	0.134	0.162	0.192	0.222	0.253

TABLE III—*Continued.*

ϵ	$\alpha = 10°$	$\alpha = 11°$	$\alpha = 12°$	$\alpha = 13°$	$\alpha = 14°$
	(*B*)	(*B*)	(*B*)	(*B*)	(*B*)
0	1.015	1.019	1.022	1.026	1.031
5	1.031	1.037	1.041	1.047	1.053
10	1.046	1.055	1.061	1.068	1.076
15	1.063	1.073	1.081	1.090	1.100
20	1.081	1.092	1.103	1.112	1.125
25	1.099	1.112	1.124	1.136	1.150
30	1.119	1.135	1.151	1.163	1.179
35	1.141	1.159	1.175	1.195	1.211
40	1.166	1.186	1.205	1.225	1.245
45	1.195	1.218	1.240	1.263	1.288
	(*C*)	(*C*)	(*C*)	(*C*)	(*C*)
	0.030	0.036	0.043	0.051	0.029

TABLE IV—*Continued.*

ϵ	$\alpha = 10°$	$\alpha = 11°$	$\alpha = 12°$	$\alpha = 13°$	$\alpha = 14°$
	(*D*)	(*D*)	(*D*)	(*D*)	(*D*)
0	0.970	0.964	0.957	0.950	0.942
5	1.000	0.997	0.993	0.988	0.983
10	1.031	1.031	1.030	1.028	1.026
15	1.064	1.067	1.069	1.061	1.072
20	1.099	1.105	1.110	1.116	1.121
25	1.136	1.147	1.156	1.165	1.173
30	1.178	1.194	1.204	1.220	1.232
35	1.224	1.244	1.262	1.281	1.300
40	1.291	1.304	1.328	1.353	1.377
45	1.342	1.375	1.407	1.438	1.469

TABLE V—*Continued.*

ϵ	$\alpha = 10°$	$\alpha = 11°$	$\alpha = 12°$	$\alpha = 13°$	$\alpha = 14°$
	(*E*)	(*E*)	(*E*)	(*E*)	(*E*)
0	0	0	0	0	0
5	0.030	0.032	0.036	0.039	0.042
10	0.061	0.067	0.073	0.079	0.085
15	0.093	0.102	0.111	0.119	0.130
20	0.124	0.137	0.150	0.163	0.175
25	0.156	0.173	0.189	0.205	0.221
30	0.188	0.208	0.216	0.248	0.269
35	0.220	0.244	0.268	0.292	0.316
40	0.252	0.280	0.308	0.336	0.365
45	0.284	0.316	0.349	0.382	0.415

TABLE III—*Continued.*

ϵ	$a = 15°$	$a = 16°$	$a = 17°$	$a = 18°$	$a = 20°$
	(*B*)	(*B*)	(*B*)	(*B*)	(*B*)
0	1.035	1.040	1.048	1.051	1.062
5	1.059	1.066	1.076	1.081	1.098
10	1.084	1.093	1.104	1.112	1.132
15	1.110	1.120	1.134	1.138	1.168
20	1.135	1.149	1.165	1.177	1.218
25	1.165	1.179	1.197	1.211	1.245
30	1.195	1.212	1.233	1.248	1.288
35	1.229	1.249	1.272	1.291	1.339
40	1.268	1.291	1.317	1.340	1.389
45	1.313	1.338	1.369	1.393	1.451
	(*C*)	(*C*)	(*C*)	(*C*)	(*C*)
	0.067	0.076	0 086	0.095	0.117

TABLE IV—*Continued.*

ϵ	$a = 15°$	$a = 16°$	$a = 17°$	$a = 18°$	$a = 20°$
	(*D*)	(*D*)	(*D*)	(*D*)	(*D*)
0	0.933	0.924	0.915	0.905	0.883
5	0.977	0.971	0.964	0.957	0.940
10	1.023	1.018	1.016	1.011	1.000
15	1.072	1.073	1.071	1 069	1.068
20	1.124	1.127	1.129	1.131	1.132
25	1.181	1.188	1.194	1.200	1.208
30	1.244	1.256	1.266	1.276	1.293
35	1.316	1.332	1.348	1.363	1.390
40	1.400	1.422	1.444	1.465	1.505
45	1.500	1.530	1.559	1.588	1.643

TABLE V—*Continued.*

ϵ	$a = 15°$	$a = 16°$	$a = 17°$	$a = 18°$	$a = 20°$
	(*E*)	(*E*)	(*E*)	(*E*)	(*E*)
0	0	0	0	0	0
5	0.045	0.047	0.050	0.053	0.058
10	0.091	0.097	0.102	0.108	0.119
15	0.139	0.148	0.157	0.165	0.183
20	0.188	0.200	0.213	0.225	0.249
25	0.238	0.254	0.270	0.177	0.318
30	0.289	0.309	0.329	0.349	0.389
35	0.341	0.365	0.390	0.414	0.463
40	0.394	0.423	0.452	0.481	0.539
45	0.448	0.482	0.516	0.551	0.620

TABLE VI.

NATURAL SINES, COSINES, TANGENTS AND COTANGENTS.

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

′	5° Sine	5° Cosin	6° Sine	6° Cosin	7° Sine	7° Cosin	8° Sine	8° Cosin	9° Sine	9° Cosin	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10193	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21730	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

′	25° Sine	25° Cosin	26° Sine	26° Cosin	27° Sine	27° Cosin	28° Sine	28° Cosin	29° Sine	29° Cosin	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

′	30° Sine	30° Cosin	31° Sine	31° Cosin	32° Sine	32° Cosin	33° Sine	33° Cosin	34° Sine	34° Cosin	′
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73088	.68878	.72497	70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	35
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74352	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

′	4°		5°		6°		7°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	8.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

′	8° Tang	8° Cotang	9° Tang	9° Cotang	10° Tang	10° Cotang	11° Tang	11° Cotang	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

′	12°		13°		14°		15°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

′	16° Tang	16° Cotang	17° Tang	17° Cotang	18° Tang	18° Cotang	19° Tang	19° Cotang	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang 73°	Tang 73°	Cotang 72°	Tang 72°	Cotang 71°	Tang 71°	Cotang 70°	Tang 70°	′

′	20° Tang	20° Cotang	21° Tang	21° Cotang	22° Tang	22° Cotang	23° Tang	23° Cotang	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

′	28°		29°		30°		31°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	61°		60°		59°		58°		

′	32° Tang	32° Cotang	33° Tang	33° Cotang	34° Tang	34° Cotang	35° Tang	35° Cotang	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	57°		56°		55°		54°		

′	36° Tang	36° Cotang	37° Tang	37° Cotang	38° Tang	38° Cotang	39° Tang	39° Cotang	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	53° Cotang	53° Tang	52° Cotang	52° Tang	51° Cotang	51° Tang	50° Cotang	50° Tang	′

′	40°		41°		42°		43°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1 10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

′	44° Tang	44° Cotang	′	′	44° Tang	44° Cotang	′	′	44° Tang	44° Cotang	′
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	45° Cotang	45° Tang	′	′	45° Cotang	45° Tang	′	′	45° Cotang	45° Tang	′

SHORT-TITLE CATALOGUE

OF THE

PUBLICATIONS

OF

JOHN WILEY & SONS,

NEW YORK.

LONDON: CHAPMAN & HALL, LIMITED.

ARRANGED UNDER SUBJECTS.

Descriptive circulars sent on application.

Books marked with an asterisk are sold at *net* prices only.

All books are bound in cloth unless otherwise stated.

AGRICULTURE.

CATTLE FEEDING—DAIRY PRACTICE—DISEASES OF ANIMALS—GARDENING, ETC.

Armsby's Manual of Cattle Feeding	12mo,	$1 75
Downing's Fruit and Fruit Trees	8vo,	5 00
Grotenfelt's The Principles of Modern Dairy Practice. (Woll.)	12mo,	2 00
Kemp's Landscape Gardening	12mo,	2 50
Lloyd's Science of Agriculture	8vo,	4 00
Loudon's Gardening for Ladies. (Downing.)	12mo,	1 50
Steel's Treatise on the Diseases of the Dog	8vo,	3 50
" Treatise on the Diseases of the Ox	8vo,	6 00
Stockbridge's Rocks and Soils	8vo,	2 50
Woll's Handbook for Farmers and Dairymen	12mo,	1 50

ARCHITECTURE.

BUILDING—CARPENTRY—STAIRS—VENTILATION, ETC.

Berg's Buildings and Structures of American Railroads	4to,	7 50
Birkmire's American Theatres—Planning and Construction	8vo,	3 00
" Architectural Iron and Steel	8vo,	3 50
Birkmire's Compound Riveted Girders	8vo,	2 00
" Skeleton Construction in Buildings	8vo,	3 00
Carpenter's Heating and Ventilating of Buildings	8vo,	3 00

Downing, Cottages	8vo,	$2 50
" Hints to Architects	8vo,	2 00
Freitag's Architectural Engineering	8vo,	2 50
Gerhard's Sanitary House Inspection	16mo,	1 00
" Theatre Fires and Panics	12mo,	1 50
Hatfield's American House Carpenter	8vo,	5 00
Holly's Carpenter and Joiner	18mo,	75
Kidder's Architect and Builder's Pocket-book	Morocco flap,	4 00
Merrill's Stones for Building and Decoration	8vo,	5 00
Monckton's Stair Building—Wood, Iron, and Stone	4to,	4 00
Stevens' House Painting	18mo,	75
Wait's Engineering and Architectural Jurisprudence.	*(In the press.)*	
Worcester's Small Hospitals—Establishment and Maintenance, including Atkinson's Suggestions for Hospital Architecture	12mo,	1 25
World's Columbian Exposition of 1893	4to,	2 50

ARMY, NAVY, Etc.

MILITARY ENGINEERING—ORDNANCE—PORT CHARGES, ETC.

Bourne's Screw Propellers	4to,	5 00
Bruff's Ordnance and Gunnery	8vo,	6 00
Bucknill's Submarine Mines and Torpedoes	8vo,	4 00
Chase's Screw Propellers	8vo,	3 00
Cooke's Naval Ordnance	8vo,	12 50
Cronkhite's Gunnery for Non-com. Officers	18mo, morocco,	2 00
De Brack's Cavalry Outpost Duties. (Carr.)	18mo, morocco,	2 00
Dietz's Soldier's First Aid	12mo, morocco,	1 25
* Dredge's Modern French Artillery	4to, half morocco,	20 00
" Record of the Transportation Exhibits Building, World's Columbian Exposition of 1893	4to, half morocco,	15 00
Dyer's Light Artillery	12mo,	3 00
Hoff's Naval Tactics	8vo,	1 50
Hunter's Port Charges	8vo, half morocco,	13 00
Ingalls's Ballistic Tables	8vo,	1 50
" Handbook of Problems in Direct Fire	8vo,	4 00
Mahan's Advanced Guard	18mo,	1 50
" Permanent Fortifications. (Mercur.)	8vo, half morocco,	7 50
Mercur's Attack of Fortified Places	12mo,	2 00

Mercur's Elements of the Art of War	8vo,	$4 00
Metcalfe's Ordnance and Gunnery	12mo, with Atlas,	5 00
Phelps's Practical Marine Surveying	8vo,	2 50
Powell's Army Officer's Examiner	12mo,	4 00
Reed's Signal Service		50
Sharpe's Subsisting Armies	18mo, morocco,	1 50
Strauss and Alger's Naval Ordnance and Gunnery		
Todd and Whall's Practical Seamanship	8vo,	7 50
Very's Navies of the World	8vo, half morocco,	3 50
Wheeler's Siege Operations	8vo,	2 00
Winthrop's Abridgment of Military Law	12mo,	2 50
Woodhull's Notes on Military Hygiene	12mo, morocco,	2 50
Young's Simple Elements of Navigation	12mo, morocco flaps,	2 50

ASSAYING.

Smelting—Ore Dressing—Alloys, Etc.

Fletcher's Quant. Assaying with the Blowpipe	12mo, morocco,	1 50
Furman's Practical Assaying	8vo,	3 00
Kunhardt's Ore Dressing	8vo,	1 50
* Mitchell's Practical Assaying. (Crookes.)	8vo,	10 00
O'Driscoll's Treatment of Gold Ores	8vo,	2 00
Ricketts and Miller's Notes on Assaying	8vo,	3 00
Thurston's Alloys, Brasses, and Bronzes	8vo,	2 50
Wilson's Cyanide Processes	12mo,	1 50
" The Chlorination Process	12mo,	1 50

ASTRONOMY.

Practical, Theoretical, and Descriptive.

Craig's Azimuth	4to,	3 50
Doolittle's Practical Astronomy	8vo,	4 00
Gore's Elements of Geodesy	8vo,	2 50
Michie and Harlow's Practical Astronomy	8vo,	3 00
White's Theoretical and Descriptive Astronomy	12mo,	2 00

BOTANY.

Gardening for Ladies, Etc.

Baldwin's Orchids of New England	8vo,	1 50
Loudon's Gardening for Ladies. (Downing.)	12mo,	1 50
Thomé's Structural Botany	18mo,	2 25
Westermaier's General Botany. (Schneider.)	8vo,	2 00

BRIDGES, ROOFS, Etc.

CANTILEVER—DRAW—HIGHWAY—SUSPENSION.

(*See also* ENGINEERING, p. 6.)

Boller's Highway Bridges	8vo,	$2 00
* " The Thames River Bridge	4to, paper,	5 00
Burr's Stresses in Bridges	8vo,	3 50
Crehore's Mechanics of the Girder	8vo,	5 00
Dredge's Thames Bridges	7 parts,	
Du Bois's Stresses in Framed Structures	4to,	10 00
Foster's Wooden Trestle Bridges	4to,	5 00
Greene's Arches in Wood, etc	8vo,	2 50
" Bridge Trusses	8vo,	2 50
" Roof Trusses	8vo,	1 25
Howe's Treatise on Arches	8vo,	4 00
Johnson's Modern Framed Structures	4to,	10 00
Merriman & Jacoby's Text-book of Roofs and Bridges. Part I., Stresses	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges. Part II., Graphic Statics	8vo,	2 50
Merriman & Jacoby's Text-book of Roofs and Bridges. Part III., Bridge Design	8vo,	5 00
Merriman & Jacoby's Text-book of Roofs and Bridges. Part IV., Continuous, Draw, Cantilever, Suspension, and Arched Bridges	(*In preparation*).	
* Morison's The Memphis Bridge	Oblong 4to,	10 00
Waddell's Iron Highway Bridges	8vo,	4 00
Wood's Construction of Bridges and Roofs	8vo,	2 00
Wright's Designing of Draw Spans	8vo,	

CHEMISTRY.

QUALITATIVE—QUANTITATIVE—ORGANIC—INORGANIC, ETC.

Adriance's Laboratory Calculations	12mo,	1 25
Allen's Tables for Iron Analysis	8vo,	3 00
Austen's Notes for Chemical Students	12mo,	1 50
Bolton's Student's Guide in Quantitative Analysis	8vo,	1 50
Classen's Analysis by Electrolysis. (Herrick.)	8vo,	3 00
Crafts's Qualitative Analysis. (Schaeffer.)	12mo,	1 50
Drechsel's Chemical Reactions. (Merrill.)	12mo,	1 25
Fresenius's Quantitative Chemical Analysis. (Allen.)	8vo,	6 00

Fresenius's Qualitative Chemical Analysis. (Johnson.)....8vo, $4 00
Fuerte's Water and Public Health....................12mo, 1 50
Gill's Gas and Fuel Analysis............................12mo, 1 25
Hammarsten's Physiological Chemistry. (Mandel.)........8vo, 4 00
Helm's Principles of Mathematical Chemistry. (Morgan).12mo, 1 50
Kolbe's Inorganic Chemistry..........................12mo, 1 50
Landauer's Spectral Analysis. (Tingle.).........(*In the press*).
Mandel's Bio-chemical Laboratory......................12mo, 1 50
Mason's Water-supply....................................8vo, 5 00
Miller's Chemical Physics...............................8vo, 2 00
Mixter's Elementary Text-book of Chemistry............12mo, 1 50
Morgan's The Theory of Solutions and its Results.......12mo, 1 00
Nichols's Water Supply (Chemical and Sanitary)..........8vo, 2 50
O'Brine's Laboratory Guide to Chemical Analysis.........8vo, 2 00
Perkins's Qualitative Analysis..........................12mo, 1 00
Pinner's Organic Chemistry. (Austen.)................12mo, 1 50
Ricketts and Russell's Notes on Inorganic Chemistry (Non-metallic).....................Oblong 8vo, morocco, 75
Schimpf's Volumetric Analysis........................12mo, 2 50
Spencer's Sugar Manufacturer's Handbook.12mo, morocco flaps, 2 00
" Handbook for Chemists of Beet Sugar House. (*In the press*).
Stockbridge's Rocks and Soils...........................8vo, 2 50
Troilius's Chemistry of Iron............................8vo, 2 00
Wiechmann's Chemical Lecture Notes...................12mo, 3 00
" Sugar Analysis...............................8vo, 2 50
Wulling's Inorganic Phar. and Med. Chemistry..........12mo, 2 00

DRAWING.

ELEMENTARY—GEOMETRICAL—TOPOGRAPHICAL.

Hill's Shades and Shadows and Perspective...............8vo, 2 00
MacCord's Descriptive Geometry..........................8vo, 3 00
" Kinematics....................................8vo, 5 00
" Mechanical Drawing...........................8vo, 4 00
Mahan's Industrial Drawing. (Thompson.)........2 vols., 8vo, 3 50
Reed's Topographical Drawing. (H. A.)..................4to, 5 00
Smith's Topographical Drawing. (Macmillan.)............8vo, 2 50
Warren's Descriptive Geometry...................2 vols., 8vo, 3 50
" Drafting Instruments..........................12mo, 1 25

Warren's Free-hand Drawing	12mo,	$1 00
" Higher Linear Perspective	8vo,	3 50
" Linear Perspective	12mo,	1 00
" Machine Construction	2 vols., 8vo,	7 50
" Plane Problems	12mo,	1 25
" Primary Geometry	12mo,	75
" Problems and Theorems	8vo,	2 50
" Projection Drawing	12mo,	1 50
" Shades and Shadows	8vo,	3 00
" Stereotomy—Stone Cutting	8vo,	2 50
Whelpley's Letter Engraving	12mo,	2 00

ELECTRICITY AND MAGNETISM.

ILLUMINATION—BATTERIES—PHYSICS.

Anthony and Brackett's Text-book of Physics (Magie).	8vo,	4 00
Barker's Deep-sea Soundings	8vo,	2 00
Benjamin's Voltaic Cell	8vo,	3 00
Cosmic Law of Thermal Repulsion	18mo,	75
Crehore and Squier's Experiments with a New Polarizing Photo-Chronograph	8vo,	3 00
* Dredge's Electric Illuminations.	2 vols., 4to, half morocco,	25 00
" " " Vol. II	4to,	7 50
Gilbert's De magnete. (Mottelay.)	8vo,	2 50
Holman's Precision of Measurements	8vo,	2 00
Michie's Wave Motion Relating to Sound and Light,	8vo,	4 00
Morgan's, The Theory of Solutions and its Results	12mo,	1 00
Niaudet's Electric Batteries. (Fishback.)	12mo,	2 50
Reagan's Steam and Electrical Locomotives	12mo	2 00
Thurston's Stationary Steam Engines for Electric Lighting Purposes	12mo,	1 50
Tillman's Heat	8vo,	1 50

ENGINEERING.

CIVIL—MECHANICAL—SANITARY, ETC.

(*See also* BRIDGES, p. 4; HYDRAULICS, p. 8; MATERIALS OF ENGINEERING, p. 9; MECHANICS AND MACHINERY, p. 11; STEAM ENGINES AND BOILERS, p. 14.)

Baker's Masonry Construction	8vo,	5 00
Baker's Surveying Instruments	12mo,	3 00
Black's U. S. Public Works	4to,	5 00

Butts's Engineer's Field-book	12mo, morocco,	$2 50
Byrne's Highway Construction	8vo,	7 50
Carpenter's Experimental Engineering	8vo,	6 00
Church's Mechanics of Engineering—Solids and Fluids	8vo,	6 00
" Notes and Examples in Mechanics	8vo,	2 00
Crandall's Earthwork Tables	8vo,	1 50
Crandall's The Transition Curve	12mo, morocco,	1 50
* Dredge's Penn. Railroad Construction, etc.	Folio, half mor.,	20 00
* Drinker's Tunnelling	4to, half morocco,	25 00
Eissler's Explosives—Nitroglycerine and Dynamite	8vo,	4 00
Gerhard's Sanitary House Inspection	16mo,	1 00
Godwin's Railroad Engineer's Field-book	12mo, pocket-bk. form,	2 50
Gore's Elements of Goodesy	8vo,	2 50
Howard's Transition Curve Field-book	12mo, morocco flap,	1 50
Howe's Retaining Walls (New Edition.)	12mo,	1 25
Hudson's Excavation Tables. Vol. II	8vo,	1 00
Hutton's Mechanical Engineering of Power Plants	8vo,	5 00
Johnson's Materials of Construction	8vo,	6 00
Johnson's Stadia Reduction Diagram	Sheet, $22\frac{1}{2} \times 28\frac{1}{2}$ inches,	50
" Theory and Practice of Surveying	8vo,	4 00
Kent's Mechanical Engineer's Pocket-book	12mo, morocco,	5 00
Kiersted's Sewage Disposal	12mo,	1 25
Kirkwood's Lead Pipe for Service Pipe	8vo,	1 50
Mahan's Civil Engineering. (Wood.)	8vo,	5 00
Merriman and Brook's Handbook for Surveyors	12mo, mor.,	2 00
Merriman's Geodetic Surveying	8vo,	2 00
" Retaining Walls and Masonry Dams	8vo,	2 00
Mosely's Mechanical Engineering. (Mahan.)	8vo,	5 00
Nagle's Manual for Railroad Engineers	12mo, morocco,	
Patton's Civil Engineering	8vo,	7 50
" Foundations	8vo,	5 00
Rockwell's Roads and Pavements in France	12mo,	1 25
Ruffner's Non-tidal Rivers	8vo,	1 25
Searles's Field Engineering	12mo, morocco flaps,	3 00
Searles's Railroad Spiral	12mo, morocco flaps,	1 50
Siebert and Biggin's Modern Stone Cutting and Masonry	8vo,	1 50
Smith's Cable Tramways	4to,	2 50
" Wire Manufacture and Uses	4to,	3 00

Spalding's Roads and Pavements	12mo,	$2 00
" Hydraulic Cement	12mo,	2 00
Thurston's Materials of Construction	8vo,	5 00
* Trautwine's Civil Engineer's Pocket-book	12mo, mor. flaps,	5 00
* " Cross-section	Sheet,	25
* " Excavations and Embankments	8vo,	2 00
* " Laying Out Curves	12mo, morocco,	2 50
Wait's Engineering and Architectural Jurisprudence.	*(In the press.)*	
Warren's Stereotomy—Stone Cutting	8vo,	2 50
Webb's Engineering Instruments	12mo, morocco,	1 00
Wegmann's Construction of Masonry Dams	4to,	5 00
Wellington's Location of Railways	8vo,	5 00
Wheeler's Civil Engineering	8vo,	4 00
Wolff's Windmill as a Prime Mover	8vo,	3 00

HYDRAULICS.

WATER-WHEELS—WINDMILLS—SERVICE PIPE—DRAINAGE, ETC.

(*See also* ENGINEERING, p. 6.)

Bazin's Experiments upon the Contraction of the Liquid Vein (Trautwine)	8vo,	2 00
Bovey's Treatise on Hydraulics	8vo,	4 00
Coffin's Graphical Solution of Hydraulic Problems	12mo,	2 50
Ferrel's Treatise on the Winds, Cyclones, and Tornadoes	8vo,	4 00
Fuerte's Water and Public Health	12mo,	1 50
Ganguillet & Kutter's Flow of Water. (Hering & Trautwine.)	8vo,	4 00
Hazen's Filtration of Public Water Supply	8vo,	2 00
Herschel's 115 Experiments	8vo,	2 00
Kiersted's Sewage Disposal	12mo,	1 25
Kirkwood's Lead Pipe for Service Pipe	8vo,	1 50
Mason's Water Supply	8vo,	5 00
Merriman's Treatise on Hydraulics	8vo,	4 00
Nichols's Water Supply (Chemical and Sanitary)	8vo,	2 50
Ruffner's Improvement for Non-tidal Rivers	8vo,	1 25
Wegmann's Water Supply of the City of New York	4to,	10 00
Weisbach's Hydraulics. (Du Bois.)	8vo,	5 00
Wilson's Irrigation Engineering	8vo,	4 00
Wolff's Windmill as a Prime Mover	8vo,	3 00
Wood's Theory of Turbines	8vo,	2 50

MANUFACTURES.

ANILINE—BOILERS—EXPLOSIVES—IRON—SUGAR—WATCHES—WOOLLENS, ETC.

Allen's Tables for Iron Analysis	8vo,	$3 00
Beaumont's Woollen and Worsted Manufacture	12mo,	1 50
Bolland's Encyclopædia of Founding Terms	12mo,	3 00
" The Iron Founder	12mo,	2 50
" " " " Supplement	12mo,	2 50
Booth's Clock and Watch Maker's Manual	12mo,	2 00
Bouvier's Handbook on Oil Painting	12mo,	2 00
Eissler's Explosives, Nitroglycerine and Dynamite	8vo,	4 00
Ford's Boiler Making for Boiler Makers	18mo,	1 00
Metcalfe's Cost of Manufactures	8vo,	5 00
Metcalf's Steel—A Manual for Steel Users	12mo,	2 00
Reimann's Aniline Colors. (Crookes.)	8vo,	2 50
* Reisig's Guide to Piece Dyeing	8vo,	25 00
Spencer's Sugar Manufacturer's Handbook	12mo, mor. flap,	2 00
" Handbook for Chemists of Beet Houses. (*In the press.*)		
Svedelius's Handbook for Charcoal Burners	12mo,	1 50
The Lathe and Its Uses	8vo,	6 00
Thurston's Manual of Steam Boilers	8vo,	5 00
Walke's Lectures on Explosives	8vo.	4 00
West's American Foundry Practice	12mo,	2 50
" Moulder's Text-book	12mo,	2 50
Wiechmann's Sugar Analysis	8vo,	2 50
Woodbury's Fire Protection of Mills	8vo,	2 50

MATERIALS OF ENGINEERING.

STRENGTH—ELASTICITY—RESISTANCE, ETC.

(*See also* ENGINEERING, p. 6.)

Baker's Masonry Construction	8vo,	5 00
Beardslee and Kent's Strength of Wrought Iron	8vo,	1 50
Bovey's Strength of Materials	8vo,	7 50
Burr's Elasticity and Resistance of Materials	8vo,	5 00
Byrne's Highway Construction	8vo,	5 00
Carpenter's Testing Machines and Methods of Testing Materials		
Church's Mechanic's of Engineering—Solids and Fluids	8vo,	6 00
Du Bois's Stresses in Framed Structures	4to,	10 00

Hatfield's Transverse Strains	8vo,	$5 00
Johnson's Materials of Construction	8vo,	6 00
Lanza's Applied Mechanics	8vo,	7 50
" Strength of Wooden Columns	8vo, paper,	50
Merrill's Stones for Building and Decoration	8vo,	5 00
Merriman's Mechanics of Materials	8vo,	4 00
Patton's Treatise on Foundations	8vo,	5 00
Rockwell's Roads and Pavements in France	12mo,	1 25
Spalding's Roads and Pavements	12mo,	2 00
Thurston's Materials of Construction	8vo,	5 00
Thurston's Materials of Engineering	3 vols., 8vo,	8 00
Vol. I., Non-metallic	8vo,	2 00
Vol. II., Iron and Steel	8vo,	3 50
Vol. III., Alloys, Brasses, and Bronzes	8vo,	2 50
Weyrauch's Strength of Iron and Steel. (Du Bois.)	8vo,	1 50
Wood's Resistance of Materials	8vo,	2 00

MATHEMATICS.

CALCULUS—GEOMETRY—TRIGONOMETRY, ETC.

Baker's Elliptic Functions	8vo,	1 50
Ballard's Pyramid Problem	8vo,	1 50
Barnard's Pyramid Problem	8vo,	1 50
Bass's Differential Calculus	12mo,	4 00
Brigg's Plane Analytical Geometry	12mo,	1 00
Chapman's Theory of Equations	12mo,	1 50
Chessin's Elements of the Theory of Functions		
Compton's Logarithmic Computations	12mo,	1 50
Craig's Linear Differential Equations	8vo,	5 00
Davis's Introduction to the Logic of Algebra	8vo,	1 50
Halsted's Elements of Geometry	8vo,	1 75
" Synthetic Geometry	8vo,	1 50
Johnson's Curve Tracing	12mo,	1 00
" Differential Equations—Ordinary and Partial	8vo,	3 50
" Integral Calculus	12mo,	1 50
" Least Squares	12mo,	1 50
Ludlow's Logarithmic and Other Tables. (Bass.)	8vo,	2 00
" Trigonometry with Tables. (Bass.)	8vo,	3 00
Mahan's Descriptive Geometry (Stone Cutting)	8vo,	1 50

Merriman and Woodward's Higher Mathematics	8vo,	$5 00
Merriman's Method of Least Squares	8vo,	2 00
Parker's Quadrature of the Circle	8vo,	2 50
Rice and Johnson's Differential and Integral Calculus,	2 vols. in 1, 12mo,	2 50
" Differential Calculus	8vo,	3 50
" Abridgment of Differential Calculus	8vo,	1 50
Searles's Elements of Geometry	8vo,	1 50
Totten's Metrology	8vo,	2 50
Warren's Descriptive Geometry	2 vols., 8vo,	3 50
" Drafting Instruments	12mo,	1 25
" Free-hand Drawing	12mo,	1 00
" Higher Linear Perspective	8vo,	3 50
" Linear Perspective	12mo,	1 00
" Primary Geometry	12mo,	75
" Plane Problems	12mo,	1 25
" Plane Problems	12mo,	1 25
" Problems and Theorems	8vo,	2 50
" Projection Drawing	12mo,	1 50
Wood's Co-ordinate Geometry	8vo,	2 00
" Trigonometry	12mo,	1 00
Woolf's Descriptive Geometry	Royal 8vo,	3 00

MECHANICS—MACHINERY.

TEXT-BOOKS AND PRACTICAL WORKS.

(*See also* ENGINEERING, p. 6.)

Baldwin's Steam Heating for Buildings	12mo,	2 50
Benjamin's Wrinkles and Recipes	12mo,	2 00
Carpenter's Testing Machines and Methods of Testing Materials	8vo,	
Chordal's Letters to Mechanics	12mo,	2 00
Church's Mechanics of Engineering	8vo,	6 00
" Notes and Examples in Mechanics	8vo,	2 00
Crehore's Mechanics of the Girder	8vo,	5 00
Cromwell's Belts and Pulleys	12mo,	1 50
" Toothed Gearing	12mo,	1 50
Compton's First Lessons in Metal Working	12mo,	1 50
Dana's Elementary Mechanics	12mo,	1 50

Dingey's Machinery Pattern Making	12mo,	$2 00
Dredge's Trans. Exhibits Building, World Exposition,	4to, half morocco,	15 00
Du Bois's Mechanics. Vol. I., Kinematics	8vo,	3 50
" " Vol. II., Statics	8vo,	4 00
" " Vol. III., Kinetics	8vo,	3 50
Fitzgerald's Boston Machinist	18mo,	1 00
Flather's Dynamometers	12mo,	2 00
" Rope Driving	12mo,	2 00
Hall's Car Lubrication	12mo,	1 00
Holly's Saw Filing	18mo,	75
Lanza's Applied Mechanics	8vo,	7 50
MacCord's Kinematics	8vo,	5 00
Merriman's Mechanics of Materials	8vo,	4 00
Metcalfe's Cost of Manufactures	8vo,	5 00
Michie's Analytical Mechanics	8vo,	4 00
Mosely's Mechanical Engineering. (Mahan.)	8vo,	5 00
Richards's Compressed Air	12mo,	1 50
Robinson's Principles of Mechanism	8vo,	3 00
Smith's Press-working of Metals	8vo,	3 00
The Lathe and Its Uses	8vo,	6 00
Thurston's Friction and Lost Work	8vo,	3 00
" The Animal as a Machine	12mo,	1 00
Warren's Machine Construction	2 vols., 8vo,	7 50
Weisbach's Hydraulics and Hydraulic Motors. (Du Bois.)	8vo,	5 00
" Mechanics of Engineering. Vol. III., Part I., Sec. I. (Klein.)	8vo,	5 00
Weisbach's Mechanics of Engineering Vol. III., Part I., Sec. II. (Klein.)	8vo,	5 00
Weisbach's Steam Engines. (Du Bois.)	8vo,	5 00
Wood's Analytical Mechanics	8vo,	3 00
" Elementary Mechanics	12mo,	1 25
" " " Supplement and Key		1 25

METALLURGY.

IRON—GOLD—SILVER—ALLOYS, ETC.

Allen's Tables for Iron Analysis	8vo,	3 00
Egleston's Gold and Mercury	8vo,	7 50

Egleston's Metallurgy of Silver	8vo,	$7 50
* Kerl's Metallurgy—Copper and Iron	8vo,	15 00
* " " Steel, Fuel, etc	8vo,	15 00
Kunhardt's Ore Dressing in Europe	8vo,	1 50
Metcalf Steel—A Manual for Steel Users	12mo,	2 00
O'Driscoll's Treatment of Gold Ores	8vo,	2 00
Thurston's Iron and Steel	8vo,	3 50
" Alloys	8vo,	2 50
Wilson's Cyanide Processes	12mo,	1 50

MINERALOGY AND MINING.

MINE ACCIDENTS—VENTILATION—ORE DRESSING, ETC.

Barringer's Minerals of Commercial Value	*(In the pre s.)*	
Beard's Ventilation of Mines	12mo,	2 50
Boyd's Resources of South Western Virginia	8vo,	3 00
" Map of South Western Virginia	Pocket-book form,	2 00
Brush and Penfield's Determinative Mineralogy	8vo,	3 50
Chester's Catalogue of Minerals	8vo,	1 25
" Dictionary of the Names of Minerals	8vo,	3 00
Dana's American Localities of Minerals	8vo,	1 00
" Descriptive Mineralogy. (E. S.)	8vo, half morocco,	12 50
" Mineralogy and Petrography. (J. D.)	12mo,	2 00
" Minerals and How to Study Them. (E. S.)	12mo,	1 50
" Text-book of Mineralogy. (E. S.)	8vo,	3 50
*Drinker's Tunnelling, Explosives, Compounds, and Rock Drills.	4to, half morocco,	25 00
Egleston's Catalogue of Minerals and Synonyms	8vo,	2 50
Eissler's Explosives—Nitroglycerine and Dynamite	8vo,	4 00
Goodyear's Coal Mines of the Western Coast	12mo,	2 50
Hussak's Rock-forming Minerals. (Smith.)	8vo,	2 00
Ihlseng's Manual of Mining	8vo,	4 00
Kunhardt's Ore Dressing in Europe	8vo,	1 50
O'Driscoll's Treatment of Gold Ores	8vo,	2 00
Rosenbusch's Microscopical Physiography of Minerals and Rocks. (Iddings.)	8vo,	5 00
Sawyer's Accidents in Mines	8vo,	7 00
Stockbridge's Rocks and Soils	8vo,	2 50

Williams's Lithology	8vo,	$3 00
Wilson's Mine Ventilation	16mo,	1 25

STEAM AND ELECTRICAL ENGINES, BOILERS, Etc.

STATIONARY—MARINE—LOCOMOTIVE—GAS ENGINES, ETC.

(*See also* ENGINEERING, p. 6.)

Baldwin's Steam Heating for Buildings	12mo,	2 50
Clerk's Gas Engine	12mo,	4 00
Ford's Boiler Making for Boiler Makers	18mo,	1 00
Hemenway's Indicator Practice	12mo,	2 00
Hoadley's Warm-blast Furnace	8vo,	1 50
Kneass's Practice and Theory of the Injector	8vo,	1 50
MacCord's Slide Valve	8vo,	
* Maw's Marine Engines	Folio, half morocco,	18 00
Meyer's Modern Locomotive Construction	4to,	10 00
Peabody and Miller's Steam Boilers	8vo,	4 00
Peabody's Tables of Saturated Steam	8vo,	1 00
" Thermodynamics of the Steam Engine	8vo,	5 00
" Valve Gears for the Steam-Engine	8vo,	2 50
Pray's Twenty Years with the Indicator	Royal 8vo,	2 50
Pupin and Osterberg's Thermodynamics	12mo,	1 25
Reagan's Steam and Electrical Locomotives	12mo,	2 00
Röntgen's Thermodynamics. (Du Bois.)	8vo,	5 00
Sinclair's Locomotive Running	12mo,	2 00
Thurston's Boiler Explosion	12mo,	1 50
" Engine and Boiler Trials	8vo,	5 00
" Manual of the Steam Engine. Part I., Structure and Theory	8vo,	7 50
" Manual of the Steam Engine. Part II., Design, Construction, and Operation	8vo,	7 50
	2 parts,	12 00
" Philosophy of the Steam Engine	12mo,	75
" Reflection on the Motive Power of Heat. (Carnot.)	12mo,	2 00
" Stationary Steam Engines	12mo,	1 50
" Steam-boiler Construction and Operation	8vo,	5 00
Spangler's Valve Gears	8vo,	2 50

Trowbridge's Stationary Steam Engines ....4to, boards,	$2	50
Weisbach's Steam Engine. (Du Bois.)..................8vo,	5	00
Whitham's Constructive Steam Engineering................8vo,	10	00
" Steam-engine Design 8vo,	6	00
Wilson's Steam Boilers. (Flather.)....................12mo,	2	50
Wood's Thermodynamics, Heat Motors, etc...............8vo,	4	00

TABLES, WEIGHTS, AND MEASURES.

FOR ACTUARIES, CHEMISTS, ENGINEERS, MECHANICS—METRIC TABLES, ETC.

Adriance's Laboratory Calculations 12mo,	1	25
Allen's Tables for Iron Analysis.........................8vo,	3	00
Bixby's Graphical Computing Tables......... ....Sheet,		25
Compton's Logarithms...................................12mo,	1	50
Crandall's Railway and Earthwork Tables..... ....8vo,	1	50
Egleston's Weights and Measures.......................18mo,		75
Fisher's Table of Cubic Yards.................... Cardboard,		25
Hudson's Excavation Tables. Vol. II..... ....8vo,	1	00
Johnson's Stadia and Earthwork Tables 8vo,	1	25
Ludlow's Logarithmic and Other Tables. (Bass.) 12mo,	2	00
Thurston's Conversion Tables...........................8vo,	1	00
Totten's Metrology 8vo,	2	50

VENTILATION.

STEAM HEATING—HOUSE INSPECTION—MINE VENTILATION.

Baldwin's Steam Heating................................12mo,	2	50
Beard's Ventilation of Mines 12mo,	2	50
Carpenter's Heating and Ventilating of Buildings..........8vo,	3	00
Gerhard's Sanitary House Inspection ....Square 16mo,	1	00
Mott's The Air We Breathe, and Ventilation 16mo,	1	00
Reid's Ventilation of American Dwellings 12mo,	1	50
Wilson's Mine Ventilation...............................16mo,	1	25

MISCELLANEOUS PUBLICATIONS.

Alcott's Gems, Sentiment, Language..............Gilt edges,	5	00
Bailey's The New Tale of a Tub..........................8vo,		75
Ballard's Solution of the Pyramid Problem...............8vo,	1	50
Barnard's The Metrological System of the Great Pyramid..8vo,	1	50

Davis' Elements of Law	8vo,	$2 00
Emmon's Geological Guide-book of the Rocky Mountains	8vo,	1 50
Ferrel's Treatise on the Winds	8vo,	4 00
Haines' Addresses Delivered before the Am. Ry. Assn.	12mo. (*In the press.*)	
Mott's The Fallacy of the Present Theory of Sound	Sq. 16mo,	1 00
Perkins's Cornell University	Oblong 4to,	1 50
Ricketts's History of Rensselaer Polytechnic Institute	8vo,	3 00
Rotherham's The New Testament Critically Emphasized.	12mo,	1 50
Totten's An Important Question in Metrology	8vo,	2 50
Whitehouse's Lake Mœris	Paper,	25
* Wiley's Yosemite, Alaska, and Yellowstone	4to,	3 00

HEBREW AND CHALDEE TEXT-BOOKS.

For Schools and Theological Seminaries.

Gesenius's Hebrew and Chaldee Lexicon to Old Testament. (Tregelles.)	Small 4to, half morocco,	5 00
Green's Elementary Hebrew Grammar	12mo,	1 25
" Grammar of the Hebrew Language (New Edition)	8vo,	3 00
" Hebrew Chrestomathy	8vo,	2 00
Letteris's Hebrew Bible (Massoretic Notes in English).	8vo, arabesque,	2 25
Luzzato's Grammar of the Biblical Chaldaic Language and the Talmud Babli Idioms	12mo,	1 50

MEDICAL.

Bull's Maternal Management in Health and Disease	12mo,	1 00
Hammarsten's Physiological Chemistry. (Mandel.)	8vo,	4 00
Mott's Composition, Digestibility, and Nutritive Value of Food.	Large mounted chart,	1 25
Ruddiman's Incompatibilities in Prescriptions	(*In the press.*)	
Steel's Treatise on the Diseases of the Ox	8vo,	6 00
" Treatise on the Diseases of the Dog	8vo,	3 50
Worcester's Small Hospitals—Establishment and Maintenance, including Atkinson's Suggestions for Hospital Architecture	12mo,	1 25